MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

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Professor H. H. TURNER, D.Sc., F.R.S., PRESIDENT, in the Chair.

William Allan, M.A., B.Sc., 88 Leamington Terrace, Edinburgh ;
Ernest Cuthbert Atkinson, M.A., Erwood, Beckenham, Kent ;
Colonel Arthur Henry Bagnold, R.E., Warren Wood, Shooters' Hill, S.E. ;
Rev. D. B. Marsh, D.Sc., Hamilton, Ontario, Canada ; and
William Newbold, 7 Broadwater Down, Tunbridge Wells,

were balloted for and duly elected Fellows of the Society.

The following candidate was proposed for election as a Fellow of the Society, the name of the proposer from personal knowledge being appended :

William Edward Raymond, Astronomical Observer, Sydney Observatory, New South Wales, Australia (proposed by H. C. Russell).

Sixty presents were announced as having been received since the last meeting, including, amongst others :

Galilei, Opere, Edizione Nazionale, vol. xiv., presented by the Italian Government ; H. Draper and G. W. Ritchey, Construction

of a Silvered Glass Reflector, and on the Modern Reflecting Telescope and the Making and Testing of Optical Mirrors ; and N. S. Shaler, Comparison of the Features of the Earth and the Moon, presented by the Smithsonian Institution ; Porträtgallerie der Astronomischen Gesellschaft, presented by H. W. Tullberg ; E. T. Whittaker, a Treatise on Analytical Dynamics, presented by the author.

Twenty charts of the Astrographic Chart, presented by the Royal Observatory, Greenwich ; spectrograms on the rotation of the planets, &c. (six prints), presented by Percival Lowell.

On the Completion of the Solution of the Main Problem in the New Lunar Theory. By Ernest W. Brown, Sc.D., F.R.S.

The completion of a laborious piece of work which has occupied many years for its execution furnishes a suitable opportunity for giving a general account of the object for which it was undertaken and of the methods by which the results have been obtained. The problem under consideration was that of the motion of the Moon as deduced solely from the Newtonian law of gravitation. It is limited, in the first instance, to the solution of an ideal problem in which the bodies are considered as particles, and two of them move in fixed elliptic orbits round one another. This constitutes the "main problem." The history of the attempts to obtain a solution with sufficient accuracy is well known, and I shall only touch on that portion of it which is directly connected with the new theory.

The original idea of the method here adopted to obtain a complete solution—as, indeed, of nearly all the methods of those who followed—is due to Euler. The pioneer work done by him on the lunar problem has, in my opinion, never received the full credit which it deserves. This may, perhaps, be partly due to the way in which he set forth his ideas ; but it is, I think, mainly owing to the fact that his work was immediately followed by that of Laplace, whose justly great reputation in every department of mathematics, and especially in celestial mechanics, has overshadowed the claims of his predecessor. However this may be, Euler fully recognised the importance of the special method under consideration. In the introduction to a paper published in 1768, "Réflexions sur la Variation de la Lune" (*Hist. Mem. Berlin Acad. Sc.* 1766, pp. 334–353), he states the problem to be considered in the paper as follows : "Déterminer le mouvement d'une Lune qui feroit ses révolutions autour de la Terre dans le plan de l'écliptique et dont l'excentricité seroit nulle, pendant

que le Soleil se mouvroit uniformément dans un cercle autour de la Terre." After some general remarks he writes : "Quelque chimérique cette question j'ose assurer que, si l'on réussissoit à en trouver une solution parfaite on ne trouveroit presque plus de difficulté pour déterminer le vrai mouvement de la Lune réelle. Cette question est donc de la dernière importance et il sera toujours bon d'en approfondir toutes les difficultés, avant qu'on en puisse espérer une solution complète." He then proceeds to find the solution, now known as the "variation orbit," as far as the fourth power of the only small parameter present. One may almost see in the few lines just quoted a germ of the magnificent work done by Poincaré on periodic orbits within the last twenty years.

The development of this idea of Euler is mainly due to G. W. Hill, who put the earlier steps into such a form that high accuracy could be obtained without excessive labour. J. C. Adams had also taken it up and worked at it in a somewhat similar manner. Hill determined the variation orbit and the principal part of the mean motion of the perigee, while Adams also found the variation orbit, but by a less powerful method, and the principal part of the mean motion of the node.

Before taking up a complete treatment from this stage it was necessary to consider as carefully as possible the amount of labour which would be demanded. The working value of a method of treatment is not really tested by the closeness with which the first or second approximation will make the further approximations converge quickly to the desired degree of accuracy ; the real test is, perhaps, the ease with which the final approximation can be obtained. Here we have the essential difference between the present method and all other methods. The approximations of the latter proceed along powers of the disturbing force. Euler's idea was to approximate along powers of the other small constants present. This gives a more rapid convergence and a degree of certainty in knowing the limits of error of the final results which no other method approaches.

With this in view it was necessary to cast the equations of motion into such a form that the degree of accuracy demanded should be capable of being obtained with a reasonable amount of labour, and it must be made clear that this degree of accuracy had actually been attained when the work was completed. Precautions against errors of computation must be taken, and the results should, if possible, be expressed in such a form that comparison with those of previous theories is possible. These and other points are considered in the following paragraphs :—

First, every coefficient in longitude, latitude, and parallax which is as great as one-hundredth of a second of arc has been computed, and is accurate—apart from possible errors of calculation—to at least this amount. Hansen, indeed, gives his results to thousandths of a second, but certain of them are in

error by some tenths of a second; indeed, it was not possible to obtain all of them more accurately without much increasing the extent of his calculations. Some of Delaunay's coefficients, owing to slow convergence, are not accurate to one second of arc. As a matter of fact my results are obtained correctly to one-thousandth of a second, and there are comparatively few coefficients greater than this quantity which have not been obtained.

Second, the theory is expanded algebraically in powers of four of the five parameters, the fifth (the ratio of the mean motions of the Sun and Moon) having its numerical value substituted at the outset. The last is known with a degree of certainty which satisfies all the possible needs of the theory, and the effect of any possible change which may be made in its observed value can be easily deduced from Delaunay's purely literal theory. The chief advantage gained is due to the fact that slow convergence (perhaps divergence) occurs only along powers of this ratio, while there is little loss of theoretical interest in using its numerical value. Moreover it is not difficult to find out how many places of decimals are necessary at the outset in order to secure a given number of places in the results.

Third, exceptional precautions have been taken to avoid the occurrence of errors during the course of the work. Equations of verification have been computed, not only at the end of large masses of calculations, but at practically every step in the process; in fact each manuscript page of work has, on the average, not less than two test equations computed. The most dangerous sources of error—the omission of a whole set of terms or the use of a wrong set—was partly guarded against by a property of the method itself. A comparatively small error of this kind produces in the final results large systematic errors, which a rough comparison with the values of Delaunay and certain properties of the solution will detect without difficulty. Very searching final tests, eleven in number, are furnished by the remarkable relations known to exist between the expressions for the mean motions of the perigee and node and the constant term of the parallax. These were all completely satisfied. Finally, the work was so arranged that computers could be engaged to do considerable portions of it. Only one, Mr. I. I. Sterner, has actually been employed, but this disadvantage was counterbalanced by the very high accuracy of his work. A rough calculation of the chances of an error slipping by both of us in the work turned over to him and verified by me and through the special test equations for it gave two or three possible errors in the whole. Two such errors were actually detected by the numerous final tests, and these were, of course, traced down and corrected.

Fourth, the results originally computed for the rectangular coordinates of the Moon have been transformed to polar coordinates, and thus a direct comparison with those of Delaunay has been rendered possible. Newcomb had previously transformed

Hansen's results to the same system, so that these are also available for comparison. This comparison will appear in a following number of the *Monthly Notices*. Nearly all the differences Delaunay–Brown can be explained by slow convergence of the Delaunay series, and in most of the remaining cases the differences Hansen–Brown are very small. Unexplained disagreements between the new results and those of both the earlier theories only occurred in the cases of coefficients difficult to determine accurately by the latter methods, owing to the occurrence of very small divisors and the slowness of approximation proceeding along powers of the disturbing force.

Fifth, comparison of the new coefficients with those deduced from observation has at present only been possible to a limited extent, but in two cases—the mean movements of the perigee and node—it has been completed. The net result is very satisfactory. The differences in the annual motions of these two lines are less than three-tenths of a second of arc, and these are capable of being explained by reasonable suppositions concerning the figures of the Earth and Moon, the constants connected with these bodies not being yet known with sufficient accuracy. One of the most important coefficients—that of the principal parallactic inequality in longitude—appears to furnish a value for the solar parallax very near the mean of all the values obtained by other methods.

A few brief details about the amount of time and labour expended may not be uninteresting. From 1890 to 1895 certain classes of inequalities were calculated, but the work was only begun on a systematic plan, which involved a fresh computation of all inequalities previously found, at the beginning of 1896. Mr. Sterner began work for me in the autumn of 1897 and finished it in the spring of the present year,* though neither of us was by any means continuously engaged in calculation during that period. He spent on it, according to a carefully kept record, nearly three thousand hours, and I estimate my share as some five or six thousand hours, so that the calculations have probably occupied altogether about eight or nine thousand hours. There were about 13,000 multiplications of series made, containing some 400,000 separate products; the whole of the work required the writing of between four and five millions of digits and *plus* and *minus* signs.

Although the problem now completed constitutes by far the longer part of the whole, much remains to be done before it is advisable to proceed to the construction of tables. The problem solved is that of the Moon under the attractions of the Earth and Sun, the centre of mass of the Earth and Moon being supposed to move in a fixed elliptic orbit. There remains to be found the effect of the figures of the bodies—mainly that of the Earth—the effects of the differences of the actual motion of the

* Most of the expense has been met by grants from the Government Grant Committee of the Royal Society.

centre of mass of the Earth and Moon from fixed elliptic motion, due to the attractions of the planets ; and, the most difficult of all, the effects of the direct attractions of the planets on the Moon. There are many periodic coefficients, due to the last, larger than one-tenth of a second of arc in magnitude, and the whole subject needs a careful and extended investigation. An attempt to complete the problem by considering anew these remaining sources of disturbance has been already started. The difficulties presented appear to arise much less from intricate calculations than in the construction of a satisfactory method which will give the assurance that no sensible terms have been neglected. If a moderate degree of success attends these efforts it should be possible to discover whether, within the limits set by the observations, the motion of the Moon shows effects which cannot be traced to the direct operation of the Newtonian law of gravitation.

Haverford College : 1904 *November* 18.

Analysis of 145 *Terms in the Moon's Longitude*, 1750–1901.

By P. H. Cowell.

In this paper I give an analysis for 145 terms in the Moon's longitude. The observations used are the Greenwich meridian observations from 1847 to 1901 ; and in the case of all terms whose period exceeds two months, or whose period is equal or nearly equal to the anomalistic month or a sub-multiple of it, the analysis is extended so as to embrace the period 1750 to 1851 as well.

I have used the lunar day as the unit of time. The advantage of doing this is explained in a previous paper (vol. lxiv. May).

For the short-period analyses I have adopted auxiliary angles which complete an exact number of revolutions in an exact number of lunar days. The advantage of doing this is explained at the end of the paper above referred to. Owing, however, to the absence of observations near new moon it is not possible to adopt an auxiliary angle instead of the mean elongation D ; during a single period of analysis (400 lunar days) I have treated D as going through two revolutions in 57 lunar days, but the error has not been allowed to accumulate from period to period (vol. lxiv. December). This distinction is rendered necessary, because the analysis for sin D is not similar to that for cos D, whereas that for $\sin {}_mA_n$ is similar to that for $\cos {}_mA_n$, the angle ${}_mA_n$ being defined as completing n revolutions in m days.

TABLE I.

Auxiliary Angles and their Epochs used for Short-period Analyses.

Angles.							Epochs at which the Angles pass through the Value Zero.
							Lunar Days.
$80A_3$,	$80A_5$,	$80A_8$,					0·5
$74A_2$,	$74A_3$,	$74A_5$,	$74A_6$,	$74A_8$,	$74A_{10}$,	$74A_{16}$	0·0
$53A_2$,	$53A_4$,	$53A_6$,	$53A_8$,	$53A_{10}$,			0·0
$51A_4$,							0·0
$49A_7$,	$49A_9$,						0·0
$43A_5$,							0·0
$41A_1$,	$41A_3$,	$41A_6$,					0·0
$39A_4$,							0·0
$34A_1$,	$34A_2$,						−1·5
$31A_1$,	$31A_2$,	$31A_3$,					0·0
$29A_4$,							0·0
$28A_5$,							0·0
$27A_3$,	$27A_4$,	$27A_6$,					0·0
$23A_1$,	$23A_4$,						0·0
$22A_1$,							0·0
D,	2D,	3D,	4D,	5D,			...

Table I. gives a list of the auxiliary angles ; the mean motion of every angle is implied in its symbol ; its definition is, therefore, completed by its epoch. The second column of the table gives the interval after the mean lunar noon of 1750 September $12^d\ 9^h$, at which the auxiliary angles took the value zero. The want of symmetry about the 34-day analysis is due to its having been performed about last May, before the complete scheme of analysis had been drawn up. Its epoch was chosen at haphazard. It will be seen that the strips that I have described in the paper quoted (vol. lxiv. May) have to be arranged in sixteen different ways (and in a seventeenth way for the D analysis), so as to bring the lines representing every 80th, every 74th, &c., day successively into juxtaposition. The sums of the errors for corresponding days are then easily formed. This stage occupies one computer about fifteen minutes for one analysis for one period. The present paper contains (counting the analyses for the Airy observations) about 1000 repetitions of this process. When such an angle as ${}_{49}A_9$ is necessary to the scheme it is clearly economical of labour to use ${}_{49}A_7$ in preference to ${}_7A_1$.

TABLE II.

Short-period Terms.

Ref. No.	Argument.	Movement in One Lunar Day.	Auxiliary Angle.	Excess of Argument over Auxiliary Angle	
				at T_0.	in 400 Lunar Days $=2\theta$.
		°		°	°
1	$2g-5g'+2\omega-2\omega'$	22·175632	$_{80}A_5$	11·46	−129·7472
2	$2g-5g'+4\omega-4\omega'$	22·406154	,,	264·08	− 37·5384
3	$3g-5g'+4\omega-4\omega'$	35·929074	$_{80}A_8$	243·85	− 28·3704
4	$g-4g'+2\omega-4\omega'$	9·563141	$_{74}A_2$	160·81	− 66·6356
5	$g-4g'+2\omega-2\omega'$	9·672858	,,	293·67	− 22·7488
6	$g-4g'+4\omega-4\omega'$	9·903380	,,	186·29	+ 69·4600
7	$g+g'-2\omega+2\omega'$	14·312544	$_{74}A_3$	118·05	−112·8204
8	$g+g'-\omega+\omega'$	14·427805	,,	64·36	− 66·7160
9	$g+g'$	14·543066	,,	10·67	− 20·6116
10	$g+g'+\omega+\omega'$	14·768044	,,	89·84	+ 69·3796
11	$2g-3g'+2\omega-2\omega'$	24·215924	$_{74}A_5$	304·34	− 43·3604
12	$2g-3g'+3\omega-3\omega'$	24·331185	,,	250·65	+ 2·7440
13	$2g-3g'+4\omega-4\omega'$	24·446446	,,	196·96	+ 48·8484
14	$2g+2g'-2\omega+2\omega'$	28·855610	$_{74}A_6$	128·72	−133·4320
15	$2g+2g'+2\omega'$	29·195849	,,	154·20	+ 2·6636
16	$3g-2g'+2\omega-2\omega'$	38·758990	$_{74}A_8$	315·01	− 63·9720
17	$3g-2g'+3\omega-3\omega'$	38·874251	,,	261·32	− 17·8676
18	$3g-2g'+4\omega-2\omega'$	39·099229	,,	340·49	+ 72·1236
19	$4g-6g'+4\omega-4\omega'$	48·431848	$_{74}A_{10}$	248·68	− 86·7208
20	$4g-6g'+6\omega-6\omega'$	48·662370	,,	141·30	+ 5·4880
21	$6g-4g'+4\omega-4\omega'$	77·517980	$_{74}A_{16}$	270·02	−127·9440
22	$g-\omega+\omega'$	13·407659	$_{53}A_2$	349·30	− 70·8988
23	g	13·522920	,,	295·61	− 24·7944
24	$g+\omega-\omega'$	13·638181	,,	241·92	+ 21·3100
25	$g+2\omega-2\omega'$	13·753442	,,	188·23	+ 67·4144
26	$g+2\omega$	13·863159	,,	321·09	+111·3012
27	$2g$	27·045840	$_{53}A_4$	231·22	− 49·5888
28	$2g+\omega-\omega'$	27·161101	,,	177·53	− 3·4844
29	$2g+2\omega-2\omega'$	27·276362	,,	123·84	+ 42·6200
30	$2g+2\omega$	27·386079	,,	256·70	+ 86·5068
31	$3g$	40·568760	$_{53}A_6$	166·83	− 74·3832
32	$3g+2\omega$	40·908999	,,	192·31	+ 61·7124
33	$4g$	54·091680	$_{53}A_8$	102·44	− 99·1776
34	$4g+2\omega$	54·431919	,,	127·92	+ 36·9180
35	$4g+4\omega$	54·772158	,,	153·40	+173·0136

Ref. No.	Argument.	Movement in One Lunar Day.	Auxiliary Angle.	Excess of Argument over Auxiliary Angle	
				at T_0	in 400 Lunar Days $= 2\theta$.
		°		°	°
36	$5g$	67·614600	$_{53}A_{10}$	38·05	−123·9720
37	$5g+2\omega$	67·954839	,,	63·53	+ 12·1236
38	$2g+g'-\omega+\omega'$	27·950725	$_{51}A_4$	109·13	−113·8268
39	$2g+g'$	28·065986	,,	55·44	− 67·7224
40	$2g+g'+2\omega$	28·406225	,,	80·92	+ 68·3732
41	$4g-3g'+2\omega-2\omega'$	51·261764	$_{49}A_7$	236·41	− 66·7236
42	$4g-3g'+4\omega-4\omega'$	51·492286	,,	129·03	+ 25·4852
43	$4g-3g'+4\omega-2\omega'$	51·602003	,,	261·89	+ 69·3720
44	$5g-2g'+2\omega-2\omega'$	65·804830	$_{49}A_9$	319·50	−127·0484
45	$5g-2g'+4\omega-2\omega'$	66·145069	,,	344·98	+ 9·0472
46	$3g+g'$	41·588906	$_{43}A_5$	10·45	−108·6236
47	$3g+g'+2\omega$	41·929145	,,	35·93	+ 27·4720
48	$g-5g'+2\omega-2\omega'$	8·652712	$_{41}A_1$	246·81	− 51·1104
49	$2g-g'$	26·025694	$_{41}A_3$	240·76	−126·3080
50	$2g-g'+\omega-\omega'$	26·140955	,,	187·07	− 80·2036
51	$2g-g'+2\omega-2\omega'$	26·256216	,,	133·38	− 34·0992
52	$4g-2g'+2\omega-2\omega'$	52·281910	$_{41}A_6$	14·14	−160·4072
53	$4g-2g'+4\omega-2\omega'$	52·622149	,,	39·62	− 24·3116
54	$3g-4g'+2\omega-2\omega'$	36·718698	$_{39}A_4$	115·37	− 81·7512
55	$3g-4g'+4\omega-4\omega'$	36·949220	,,	7·99	+ 10·4576
56	$g-3g'-2\omega'$	10·352765	$_{34}A_1$	78·02	− 94·1880
57	$g-3g'+\omega-3\omega'$	10·468026	,,	24·33	− 48·0836
58	$g-3g'+2\omega-2\omega'$	10·693004	,,	103·50	+ 41·9076
59	$g-3g'+3\omega-3\omega'$	10·808265	,,	49·81	+ 88·0120
60	$2g-6g'+4\omega-4\omega'$	21·386008	$_{34}A_2$	207·00	+ 83·8152
61	$g-2g'-2\omega'$	11·372911	$_{31}A_1$	268·31	− 95·9968
62	$g-2g'$	11·482628	,,	41·17	− 52·1100
63	$g-2g'+\omega-\omega'$	11·597889	,,	347·48	− 6·0056
64	$g-2g'+2\omega-2\omega'$	11·713150	,,	293·79	+ 40·0988
65	$2g-4g'+2\omega-2\omega'$	23·195778	$_{31}A_2$	334·96	− 12·0112
66	$2g-4g'+3\omega-3\omega'$	23·311039	,,	281·27	+ 34·0932
67	$2g-4g+4\omega-4\omega'$	23·426300	,,	227·58	+ 80·1976
68	$3g-6g'+4\omega-4\omega'$	34·908928	$_{31}A_3$	268·75	+ 28·0876
69	$3g-6g'+6\omega-6\omega'$	35·139450	,,	161·37	+120·2964
70	$4g-5g'+4\omega-4\omega'$	49·451994	$_{29}A_4$	18·78	− 81·2712
71	$5g-4g'+4\omega-4\omega'$	63·995060	$_{28}A_5$	123·98	−116·2620
72	$5g-4g'+6\omega-4\omega'$	64·335299	,,	149·46	+ 19·8336

Ref. No.	Argument.	Movement in One Lunar Day.	Auxiliary Angle.	Excess of Argument over Auxiliary Angle at T_0.	Excess of Argument over Auxiliary Angle in 400 Lunar Days $=2\theta$.
73	$3g-g'$	39°.548614	$_{27}A_3$	335°.17	−180°.5544
74	$3g-g'+\omega-\omega'$	39.663875	,,	281.48	−134.4500
75	$3g-g'+2\omega-2\omega'$	39.779136	,,	227.79	− 88.3456
76	$3g-g'+2\omega$	39.888853	,,	0.65	− 44.4588
77	$3g-g'+3\omega-\omega'$	40.004114	,,	306.96	+ 1.6456
78	$4g-g'+2\omega-2\omega'$	53.302056	$_{27}A_4$	0.89	− 12.5108
79	$6g-2g'+4\omega-2\omega'$	79.667989	$_{27}A_6$	228.44	−132.8044
80	$g+2g'-2\omega+2\omega'$	15.332690	$_{23}A_1$	79.78	−127.7936
81	$g+2g'$	15.563212	,,	332.40	− 35.5848
82	$g+2g'+2\omega'$	15.672929	,,	105.26	+ 8.3020
83	$5g-6g'+6\omega-6\omega'$	62.185290	$_{23}A_4$	29.27	−169.3624
84	$g+3g'-2\omega+2\omega'$	16.352836	$_{22}A_1$	304.12	− 4.3200
85	$g-g'$	12.502774	D	53.69	− 46.1043
86	$g-g'+\omega-\omega'$	12.618035	,,	0	0
87	$g-g'+2\omega-2\omega'$	12.733296	,,	306.31	+ 46.1043
88	$2g-2g'-2\omega'$	24.895831	2D	334.52	−136.0956
89	$2g-2g'$	25.005548	,,	107.38	− 92.2086
90	$2g-2g'+\omega-\omega'$	25.120809	,,	53.69	− 46.1043
91	$2g-2g'+2\omega-2\omega'$	25.236070	,,	0	0
92	$2g-2g'+3\omega-3\omega'$	25.351331	,,	306.31	+ 46.1043
93	$2g-2g'+4\omega-2\omega'$	25.576509	,,	25.48	+136.0956
94	$3g-3g'+2\omega-2\omega'$	37.738844	3D	53.69	− 46.1043
95	$3g-3g'+3\omega-3\omega'$	37.854105	,,	0	0
96	$3g-3g'+4\omega-4\omega'$	37.969366	,,	306.31	+ 46.1043
97	$3g-3g'+4\omega-2\omega'$	38.079083	,,	79.17	+ 89.9913
98	$4g-4g'+4\omega-4\omega'$	50.472140	4D	0	0
99	$4g-4g'+6\omega-4\omega'$	50.812379	,,	25.48	+136.0956
100	$5g-5g'+4\omega-4\omega'$	62.974914	5D	53.69	− 46.1043
101	$2D-V+E$	24.597940	$_{74}A_5$	260.91	+109.4460
102	$2D-g-V+E$	11.075020	$_{34}A_1$	60.07	+194.7139
103	$2D-2V+2E$	23.959810	$_{74}A_5$	213.33	−145.8062
104	$2D-g+2V-2E$	12.989411	D	37.33	+148.5505
105	$2D-g-2V+2E$	10.436889	$_{34}A_1$	12.50	− 60.5383
106	$g-3V+3E$	11.608529	$_{31}A_1$	266.72	− 1.7497

Ref. No.	Argument.	Movement in One Lunar Day.	Auxiliary Angle.	Excess of Argument over Auxiliary Angle at T_0	Excess of Argument over Auxiliary Angle in 400 Lunar Days $=2\theta$.
107	$2D-g+3V-3E$	13°.627542	${}_{53}A_2$	322°.69	+ 17°.0543
108	$2D-E+J$	24.301919	${}_{74}A_5$	286.99	− 8.9623
109	$2D-g+E-J$	12.647302	D	323.68	+ 11.7066
110	$2D-g-E+J$	10.778999	${}_{34}A_1$	86.15	+ 76.3056
111	$g-2E+2J$	11.654618	${}_{31}A_1$	6.45	+ 16.6859
112	$2D-g+2E-2J$	13.581453	${}_{53}A_2$	222.95	− 1.3813
113	$g+2\varpi-3J+7°$	13.495528	${}_{53}A_2$	147.81	− 35.7513
114	$g-\Omega$	13.577730	${}_{53}A_2$	262.53	− 2.8702
115	$g+\Omega$	13.468110	${}_{53}A_2$	328.68	− 46.7184
116	$2g+2\omega+\Omega$	27.331269	${}_{53}A_4$	289.78	+ 64.5829

Table III.

Terms of Periods Comparable with One Year.

Ref. No.	Arg.	Unit.	Movement in 40 Lunar Days in Degrees.	Movement in 40 Lunar Days in Units.	Value at T_0 in Degrees.	Value at T_0 in Units.
117	$4g'-2\omega+2\omega'$		See No. 123.			
118	$3g'+2\omega'$	$360\div\frac{17}{6}$	126°.80619	$1-\frac{1}{6}\times 0.011930$	145°.2749	$6.8602\div 6$
119	$3g'$		See No. 128.			
120	$3g'-2\omega+2\omega'$	$360\div\frac{19}{6}$	113.19663	$1-\frac{1}{6}\times 0.025733$	119.7855	$6.3220\div 6$
121	$2g'+2\omega'$	$360\div\frac{21}{5}$	86.00036	$1+\frac{1}{5}\times 0.016688$	141.1374	$8.2330\div 5$
122	$2g'$		See No. 128.			
123	$2g'-\omega+\omega'$	$360\div\frac{14}{3}$	77.00123	$1-\frac{1}{3}\times 0.005508$	61.9615	$2.4096\div 3$
124	$2E-2J$		See No. 130.			
125	$2g'-2\omega+2\omega'$		See No. 131.			
126	$2V-2E$		See No. 135.			
127	$g'+2\omega'$	$360\div 8$	45.19453	$1+\quad 0.004323$	136.9999	3.0444
128	g'	$360\div 9$	40.80583	$1+\quad 0.020146$	4.1375	0.1034
129	$2E-2M$	$360\div\frac{47}{5}$	38.22036	$1-\frac{1}{5}\times 0.010120$	98.1768	$12.8175\div 5$
130	$E-J$	$360\div\frac{29}{3}$	37.36605	$1+\frac{1}{3}\times 0.010043$	21.4980	$1.7318\div 3$
131	$g'-\omega+\omega'$	$360\div 10$	36.19540	$1+\quad 0.005428$	57.8240	1.6062
132	$3V-4E-2°$	$360\div 10$	35.76947	$1-\quad 0.006404$	37.0854	1.0302
133	$E-2J+298°$	$360\div\frac{32}{3}$	33.92591	$1+\frac{1}{3}\times 0.015636$	237.3525	$21.0980\div 3$
134	$g'-2\omega+2\omega'$	$360\div\frac{57}{5}$	31.58497	$1+\frac{1}{5}\times 0.000954$	111.5105	$17.6558\div 5$
135	$V-E$	$360\div 14$	.52522	$1-\quad 0.007353$	47.5763	1.8502

TABLE IV.

Terms of Periods Comparable with Ten Years.

Ref. No.	Arg.	Unit.	Movement in 200 Lunar Days		Value at T_0	
			in Degrees.	in Units.	in Degrees.	in Units.
136	2ω	$360 \div \frac{16}{3}$	68°·0478	$1 + \frac{1}{3} \times 0{\cdot}02435$	25°·4894	$1{\cdot}1329 \div 3$
137	$2V - 3E - 5°$	$360 \div 7$	51·2212	$1 - 0{\cdot}00403$	346·5091	6·7377
138	$2\omega - 2\omega'$		See No. 140.			
139	$4M - 2E + 98°$		See No. 142.			
140	$\omega - \omega'$	$360 \div 16$	23·0522	$1 + 0{\cdot}02454$	306·3135	13·6139
141	$J + 9°$	$360 \div 21$	17·2007	$1 + 0{\cdot}00337$	91·1455	5·3168
142	$2M - E + 49°$	$360 \div 28$	12·9292	$1 + 0{\cdot}00560$	54·4667	4·2363
143	$2\varpi - 2J$	$360 \div 31$	11·7222	$1 + 0{\cdot}00941$	287·3482	24·7439
144	$-\Omega$	$360 \div 33$	10·9620	$1 + 0{\cdot}00485$	326·9251	29·9681
145	$-2\varpi + 3J - 7°$	$360 \div 66$	5·4785	$1 + 0{\cdot}00439$	147·7973	27·0962

Table II. gives a list of 116 short period terms. I selected for analysis those terms whose coefficients appeared from theory to exceed $0''{\cdot}1$. The only use made of theory is to give the arguments. In all cases the coefficient is deduced from observation. The argument is in the second column of the table, the first column containing a reference number. The third column gives the mean motion of the argument in one lunar day, and the fourth column the auxiliary angle whose mean motion is nearly equal to that of the argument. The fifth column gives the excess of the argument over the auxiliary angle at T_0, or the middle of my forty-fourth period of analysis (the calendar date, which I never use, is given on vol. lxiv. p. 421); in the sixth column the excess of motion in 400 lunar days of the argument over the auxiliary angle is given. I call this excess 2θ. The order of Table II. is (i.) Solar; (ii.) *Venus*; (iii.) *Jupiter*; (iv.) figure of Earth terms; for the solar terms the order of the auxiliary angles is that of Table I.; when several terms have the same auxiliary angle the slowest term is put first. Table III. gives a list of terms of period comparable with one year, and Table IV. gives a list of terms of period comparable with ten years. The order is that of mean motion. The method of analysis is explained in vol. lxiv. June. The reference numbers run consecutively with Table II.

Table V.

Mean Error of Moon's Longitude for each of 890 Columns of Forty Lunar Days each in Tenth of a Second of Arc.

	Columns										
	1 to 40.	41 to 80.	81 to 120.	121 to 160.	161 to 200.	201 to 240.	241 to 280.	281 to 320.	321 to 360.	361 to 400.	401 to 440.	441 to 480.
1	+80	+13	− 6	−40	−27	−35	−50	−66	−29	− 71	−43	−18
2	+53	+ 4	+13	−23	+ 3	−49	−32	−45	−36	− 50	+ 3	+ 2
3	+58	+ 3	+13	−10	−53	− 3	−15	−18	−40	− 21	−43	− 2
4	+52	+13	+24	(−25)	− 8	−27	−36	−41	−46	− 40	+ 5	− 7
5	+29	+15	−11	−26	− 1	−21	−21	−32	−10	− 52	−13	−11
6	+38	+10	− 3	+21	−33	−33	−29	−50	−20	− 51	+ 6	−25
7	+67	0	+ 8	−17	0	−46	−59	−36	− 8	− 60	+ 7	−18
8	+60	+12	+34	+27	−18	−25	−47	−45	−23	− 55	−18	−35
9	+59	− 5	+23	(+ 4)	−20	−20	−74	−31	−35	− 56	−27	−31
10	+43	+ 3	+11	−17	−45	−34	−57	−39	−38	− 79	−29	−30
11	+22	+37	+20	+45	−31	−32	−85	−34	−55	− 64	+11	− 3
12	+31	−20	+23	− 5	−35	−40	−60	−28	−19	− 40	− 1	−13
13	+12	+ 9	+12	− 2	−39	−59	−43	−45	−49	− 51	−31	−20
14	+ 8	+21	+34	+ 3	−34	−44	(−62)	−40	−33	− 51	−15	−24
15	+ 5	+ 3	+18	−10	−34	−34	(−63)	−52	−30	− 70	−37	−25
16	+17	+34	+ 8	− 4	− 5	−23	(−61)	−55	−59	− 67	−19	−18
17	+24	+12	+15	+13	−45	−45	−81	−31	−48	− 37	−41	+29
18	+39	+ 6	+ 3	+15	−36	−31	−53	−13	−84	− 65	−44	− 5
19	+28	+26	+18	+14	−47	−35	−54	−20	−81	− 61	−30	−12
20	+14	+ 5	− 2	+11	−19	(−34)	−54	− 3	−43	− 52	+20	− 7
21	+19	+11	+21	+10	+ 1	(−54)	−50	−33	−75	− 47	−32	−12
22	+19	+15	− 7	+12	− 4	−69	−15	−22	−17	− 57	−18	−34
23	+29	+ 6	− 4	+ 3	0	−38	−24	−21	−46	− 87	−26	−35
24	+47	+15	+ 6	−23	−17	−72	−45	−44	−34	− 57	−31	−30
25	+16	− 8	+19	− 2	−14	−35	−47	−35	−40	− 37	−36	− 9
26	+51	+22	+ 2	+24	−28	−29	−55	−30	−51	− 47	−21	−29
27	+ 7	+34	− 5	+26	−39	−17	−44	−17	−27	− 64	−14	−16
28	+ 6	+10	−14	+32	−17	−34	−21	−42	−75	−100	− 6	−10
29	+ 8	− 8	+49	−12	−21	−60	−25	−18	−47	− 64	[−14]	− 2
30	+36	+24	+42	−14	− 4	−70	−35	−26	−29	− 59	−14	− 2
31	+25	+ 5	+30	− 6	+ 3	−51	−35	−39	−39	− 71	− 4	−19
32	+30	− 1	− 1	− 8	+ 4	−39	−51	−41	−48	− 73	−23	+ 3
33	+25	+23	+ 5	− 9	−14	−29	+ 8	−36	−55	− 51	−12	−26
34	+34	+33	+ 5	−20	−19	−42	−13	−19	−57	− 50	− 4	−22
35	+31	0	+46	− 8	+ 8	−16	−21	−26	−69	− 25	−33	− 8
36	+33	+41	+43	+11	−15	−32	−32	−27	−40	− 32	− 1	− 1
37	+14	+ 4	+22	− 3	−15	−31	−25	−16	−42	− 52	− 3	−20
38	+15	+14	+19	−22	−12	−51	−25	−23	−67	− 55	−40	+12
39	+ 6	+30	+23	+32	−22	−48	−23	−47	−56	− 61	+ 4	+14
40	+17	+17	+15	+ 2	−20	−42	−13	−44	−65	− 29	−28	−13

	Columns										
	481 to 520.	521 to 560.	561 to 600.	601 to 640.	641 to 680.	681 to 720.	721 to 760.	761 to 800.	801 to 840.	841 to 880.	881 to 890.
1	−14	−25	+23	+29	− 4	+12	+14	+14	+ 1	−20	−32
2	+14	−10	+12	+ 6	− 9	+15	+21	+14	+11	−26	−21
3	−29	−37	+19	+14	− 2	+ 2	+55	+ 5	+ 1	−22	−45
4	+ 3	−51	+29	+12	+15	−15	+46	−16	+ 1	−19	−32
5	0	−47	+29	+ 4	+10	−20	+31	+10	+18	− 4	−28
6	+ 8	−35	+18	+17	+26	+15	+53	+14	+ 9	+ 1	−30
7	+21	+ 7	+40	+44	+29	+ 3	+27	+14	+28	+ 1	−49
8	+ 7	−36	+ 3	+40	+17	+12	+ 4	+35	− 9	− 3	−36
9	−10	−21	− 9	+50	+32	+21	+15	+24	+ 3	− 4	−43
10	−24	−18	+ 7	+36	− 5	+ 3	+11	−15	− 4	−24	−38
11	−22	−20	+29	+24	+10	−44	+13	− 4	− 7	−15	...
12	−16	−16	+17	+ 9	+ 1	−32	+29	− 3	−27	− 9	...
13	−10	− 4	−21	+ 8	− 7	−34	+ 8	− 1	− 8	−31	...
14	−12	+ 1	− 4	+ 1	−11	−16	+13	+ 9	− 3	−20	...
15	− 5	− 2	− 2	+17	+15	+10	+30	+20	+ 2	− 6	...
16	− 9	+ 3	+15	+37	+ 1	+25	+25	+25	+ 3	− 4	...
17	− 4	−20	0	+38	+24	− 2	+ 7	+36	+34	0	...
18	+ 6	0	+18	+43	(+17)	+17	+24	+15	+19	− 7	...
19	−12	−22	+21	+ 5	(+17)	− 4	+45	+ 1	+11	−33	...
20	−10	−27	− 1	+10	+26	+14	+21	−22	−20	−45	...
21	+10	−26	0	+ 2	+11	− 3	+ 9	− 7	−26	−36	...
22	−21	−20	+ 7	− 6	+ 4	−26	+10	−18	−11	−32	...
23	−16	−17	+40	−11	− 3	− 9	0	+ 1	+10	−47	...
24	+ 7	−23	+10	+ 4	+ 9	−14	+35	−20	− 5	−40	...
25	+ 5	−18	+15	+32	+16	− 7	+27	+24	+12	−23	...
26	+ 1	0	+24	+19	+15	− 1	+22	+11	+ 1	−10	...
27	−13	0	+26	+12	+21	+22	+14	+13	+11	−27	...
28	− 3	− 5	+17	+19	+17	+31	+36	− 5	−15	−34	...
29	+ 2	−45	+27	+14	+ 1	+ 6	+17	+10	−13	−41	...
30	−33	+ 7	+15	− 5	−11	− 7	+24	−26	−18	−23	...
31	− 6	−12	+16	−10	− 5	+26	+22	+ 1	−21	−40	...
32	−19	−17	+ 9	0	+ 3	− 6	+10	−11	−13	−12	...
33	−10	−16	+28	− 2	− 4	+18	− 6	− 3	−35	−23	...
34	−22	−12	+10	+ 9	− 1	+16	+12	+ 6	−32	−26	...
35	− 4	−15	+ 8	+12	− 8	+ 3	+ 7	+25	−10	+ 2	...
36	−46	+ 4	+16	+34	−13	+34	− 9	+24	−31	−20	...
37	−49	−22	+13	+47	+20	+14	+ 7	+ 9	+13	−36	...
38	−26	+ 7	+12	+27	− 5	+34	+16	[+15]	−43	−46	...
39	−22	− 5	−20	−14	−20	+12	+25	+ 4	−19	−26	...
40	−32	− 2	+19	+18	+23	+13	+27	+22	−16	−30	...

TABLE VI.

Mean Error of Moon's Longitude for each of 178 *Half-Periods of* 200 *Lunar Days each in Tenths of a Second of Arc.*

	1A to 8B.	9A to 16B.	17A to 24B.	25A to 32B.	33A to 40B.	41A to 48B.	49A to 56B.	57A to 64B.	65A to 72B.	73A to 80B.	81A to 88B.	89A and 89B.
1	+28	− 8	− 2	+ 8	+11	+13	+12	+24	− 9	+20	+10	+2
2	+27	0	− 8	−14	+18	+19	+17	+14	+ 9	+ 9	+ 9	−5
3	−11	+ 9	−17	−21	+ 4	+15	+ 2	+ 3	−11	+ 7	− 1	
4	− 3	− 4	−12	−19	−22	+ 7	+ 9	+10	+ 4	+12	+17	
5	− 1	− 3	+15	+ 8	− 3	− 2	+ 9	+11	− 8	+ 4	+ 7	
6	− 5	+ 5	0	+ 8	− 6	+13	+ 3	+19	− 6	+11	+ 4	
7	+ 4	+12	+21	+22	−15	+11	− 1	+ 9	−18	0	− 7	
8	− 8	+19	+ 8	+20	−15	+12	−24	+ 3	−14	+ 4	− 4	
9	−14	−26	+ 2	+ 5	−10	+17	−25	+ 8	−15	− 2	+ 2	
10	−20	+ 3	− 3	+ 5	−23	− 4	−12	+32	− 3	+ 7	+14	
11	−13	+ 8	− 9	+ 4	−19	+ 5	0	+ 6	−38	0	+ 9	
12	− 6	+12	− 1	+20	−20	+19	− 5	+21	− 5	+ 7	+ 7	
13	−13	+ 6	−19	+13	−22	− 3	−15	− 4	−27	− 6	− 8	
14	− 5	+17	− 7	+17	−32	+ 9	− 3	+ 4	− 5	− 1	+ 1	
15	− 6	0	+ 2	+11	−21	+ 4	− 9	− 8	− 4	+ 6	+11	
16	+ 3	+14	− 4	+12	−13	+16	+ 1	+12	+ 6	+17	− 1	

TABLE VII.

Mean Error of Moon's Longitude for each of 96 *Half-Periods of* 200 *Lunar Days each in Tenths of a Second of Arc.*

	86A to 93B.	94A to 101B.	102A to 109B.	110A to 117B.	118A to 125B.	126A to 133B.
1	+24	+ 4	+ 1	+26	− 1	+13
2	+15	+ 2	+ 3	+17	− 1	+ 9
3	+16	+ 2	+13	+18	− 1	+ 6
4	+13	−13	+10	+17	− 9	− 3
5	+13	− 3	+19	+ 9	+ 2	− 2
6	+ 5	−15	+15	+ 5	0	0
7	+13	−12	+ 6	− 9	0	− 9

	86A to 93B.	94A to 101B.	102A to 109B.	110A to 117B.	118A to 125B.	126A to 133B.
8	− 1	−17	+ 9	−10	+ 6	− 1
9	+ 4	−29	+ 7	−21	− 3	−11
10	+ 7	−23	+ 6	− 8	+14	−10
11	+ 5	−26	+18	−18	− 1	−13
12	+ 5	−17	+18	− 6	0	− 4
13	+ 3	−18	+11	−19	− 1	− 9
14	+ 5	−12	+20	−11	+ 8	− 2
15	+ 7	− 5	+17	− 7	+18	−10
16	+ 8	0	+14	− 3	+14	− 3

Tables V., VI., VII. give the data for the analyses of Tables III. and IV. Table V. gives for the Airy period 1750 to 1851 the mean error of the Moon's longitude for successive intervals of forty lunar days ; the table is, in fact, similar to the one on vol. lxiv. p. 685 for the Hansen period 1847–1901. Numbers in round brackets have been supplied by interpolation in the absence of observations ; numbers in square brackets have been substituted for the results of obviously erroneous observations. Tables VI. and VII. refer to the Airy and Hansen periods respectively. The unit is 0″·1 instead of 0″·02 in a similar table on vol. lxiv. p. 691. The errors here given (Tables VI. and VII. only) contain my empirical terms (vol. lxv. November), but, as my empirical terms are of longer period than the terms in Table IV., the results are not affected thereby.

TABLE VIII.

Mean Values for Periods 86 *to* 133 *of the Errors of the Moon's Longitude Multiplied by certain Factors in Tenths of a Second of Arc.*

Period.	80^s3.	80^c3.	80^s5.	80^c5.	80^s8.	80^c8.	74^s2.	74^c2.	74^s3.	74^c3.	74^s5.	74^c5.
86	− 1	− 3	− 1	− 1	+ 2	+ 1	+ 1	0	+ 5	+ 3	− 3	− 7
87	− 6	− 2	− 1	+ 1	+ 1	− 1	+ 3	0	+ 5	+ 7	0	+ 3
88	+ 2	− 16	+ 2	− 2	− 1	− 1	+ 1	+ 1	0	− 2	− 3	− 3
89	+ 10	− 17	0	− 4	− 5	− 8	− 2	− 1	− 5	− 4	− 4	+ 1
90	+ 13	− 5	− 1	− 1	− 5	+ 2	− 5	− 2	−13	− 3	+ 1	0
91	+ 14	− 4	+ 1	+ 1	+ 2	0	+ 3	+ 5	− 1	− 1	− 1	− 1
92	+ 7	+ 10	+ 4	0	0	+ 3	− 4	+ 7	0	+ 4	− 1	0
93	− 5	+ 1	− 1	− 1	− 1	− 3	+ 1	+ 2	− 2	− 1	+ 1	+ 2
94	− 3	+ 3	0	− 1	0	+ 4	− 1	− 1	+ 2	+ 2	− 6	+ 2
95	− 1	− 6	+ 2	0	0	+ 2	− 3	− 1	+ 4	− 5	− 2	− 2
96	+ 14	− 8	− 1	− 1	− 1	− 1	0	− 2	+ 6	+ 2	− 4	0
97	+ 9	0	− 2	− 1	+ 2	− 2	+ 2	+ 5	+ 3	0	− 4	+ 3
98	+ 9	+ 11	− 4	− 4	− 3	− 3	− 1	+ 1	+ 7	+ 7	− 3	+ 3
99	+ 3	+ 11	+ 6	+ 1	− 2	− 2	0	− 7	+ 6	+ 3	− 3	+ 5
100	− 8	0	− 1	+ 2	0	+ 3	− 4	+ 2	0	0	+ 3	+ 6
101	− 2	0	+ 4	− 1	+ 2	− 1	− 1	+ 1	− 6	0	− 4	− 1
102	+ 1	− 2	+ 3	− 2	+ 1	− 2	0	+ 4	− 5	− 2	+ 5	+ 3
103	+ 6	− 1	− 2	− 1	+ 1	+ 1	+ 2	0	+ 1	+ 4	− 1	+ 4
104	+ 4	+ 5	+ 2	+ 1	+ 1	− 2	0	− 1	− 4	+ 7	− 1	0
105	0	+ 10	+ 1	0	− 1	− 3	+ 5	− 1	− 9	0	− 3	− 2
106	− 5	+ 7	− 3	+ 4	+ 1	+ 1	− 1	+ 3	− 4	− 6	− 1	+ 2
107	− 7	− 2	+ 1	+ 1	0	− 3	0	+ 4	− 3	− 7	− 3	+ 6
108	− 8	− 9	− 2	− 5	− 1	+ 1	+ 5	− 1	− 1	+ 1	0	0
109	− 3	− 4	+ 2	+ 3	− 1	+ 3	+ 3	− 3	+ 1	+ 5	0	+ 3
110	+ 4	− 2	− 2	0	− 3	0	+ 1	+ 2	0	+ 2	+ 1	0
111	+ 5	− 9	+ 2	+ 3	− 1	− 1	+ 3	− 3	+ 6	− 4	− 9	+ 1
112	0	0	− 1	0	− 2	− 3	− 3	+ 3	+ 5	0	+ 1	− 3
113	− 1	+ 1	0	+ 2	+ 1	0	+ 1	+ 1	+ 5	+ 1	− 2	− 1
114	+ 2	− 9	+ 3	0	− 1	− 2	− 3	+ 5	+ 5	+ 4	+ 2	+ 2
115	+ 7	− 14	+ 1	− 1	− 2	− 4	+ 2	+ 1	+ 1	0	+ 5	0
116	+ 10	− 1	− 3	− 3	+ 1	− 3	− 2	+ 2	− 5	− 2	0	+ 1
117	+ 7	+ 1	+ 2	− 1	+ 3	+ 4	0	0	− 6	− 1	+ 1	+ 4
118	+ 10	+ 5	− 2	0	+ 2	− 1	− 2	+ 1	− 4	+ 4	+ 5	− 4
119	− 11	+ 3	+ 5	+ 2	+ 1	+ 3	+ 2	+ 5	+ 1	+ 5	+ 2	+ 1
120	− 11	− 8	+ 5	− 2	− 2	− 1	− 2	+ 2	− 4	0	− 2	+ 1
121	0	− 9	− 2	+ 2	+ 2	+ 1	− 1	− 4	− 3	− 3	− 1	− 6
122	+ 7	− 10	+ 5	− 2	− 2	+ 1	− 2	+ 2	− 5	− 7	+ 7	−11
123	+ 6	− 5	0	− 2	− 3	+ 4	+ 5	+ 1	− 1	− 2	+ 3	− 2
124	+ 11	+ 3	+ 2	− 2	0	0	− 2	+ 2	+ 4	+ 1	− 1	− 1
125	0	+ 14	− 2	− 6	− 1	+ 2	0	− 2	+ 1	+ 6	− 2	− 2
126	− 7	+ 8	− 4	+ 1	− 2	− 3	+ 4	− 4	+ 1	+ 1	− 4	0
127	− 8	+ 3	− 1	− 1	+ 2	+ 4	0	0	+ 6	+ 1	− 7	− 6
128	− 3	− 5	0	+ 1	− 2	+ 5	0	+ 2	0	0	− 7	− 7
129	+ 8	− 1	+ 3	+ 1	− 2	0	0	+ 2	0	+ 6	0	− 3
130	+ 2	+ 2	+ 2	0	0	0	0	0	− 4	+ 2	− 5	− 5
131	+ 3	+ 6	− 4	− 3	+ 2	− 1	− 1	0	− 6	+ 2	− 3	− 1
132	− 4	+ 8	0	− 1	− 1	0	0	+ 1	− 2	− 3	− 3	− 2
133	− 2	− 3	+ 2	− 4	+ 1	+ 1	− 4	+ 2	0	− 9	− 5	− 1
ms +	+174	+112	+60	+26	+28	+46	+44	+69	+75	+80	+37	+53
ms −	− 96	−155	−40	−53	−45	−51	−44	−33	−93	−62	−98	−71
neral Sum	+ 78	− 43	+20	−27	−17	− 5	0	+36	−18	+18	−61	−18

Period.	$74^{s}6$.	$74^{c}6$.	$74^{s}8$.	$74^{c}8$	$74^{s}10$.	$74^{c}10$.	$74^{s}16$.	$74^{c}16$.	$53^{s}2$.	$53^{c}2$.	$53^{s}4$.	$53^{c}4$.
86	− 5	− 2	− 4	+ 3	− 1	+ 2	− 1	+ 1	− 1	− 3	− 1	+ 3
87	0	+ 3	+ 2	+ 2	0	+ 1	+ 5	0	− 4	+ 6	− 2	+ 8
88	+ 1	+ 1	0	+ 4	− 3	+ 3	+ 2	0	− 17	0	0	− 1
89	0	− 3	+ 6	− 1	− 5	− 1	− 2	− 3	− 19	+ 3	− 4	+ 3
90	+ 2	− 1	+ 5	+ 7	0	+ 3	− 4	+ 3	− 14	+ 2	− 7	− 1
91	− 3	− 1	0	− 1	+ 2	0	0	0	− 13	+ 8	+ 3	− 4
92	+ 2	− 1	+ 2	− 1	+ 3	0	− 5	− 1	− 11	0	+ 4	− 6
93	− 1	− 1	0	− 1	− 2	+ 2	− 1	0	− 2	− 5	+ 2	0
94	− 2	+ 2	− 3	− 3	+ 3	− 1	− 3	− 2	− 4	− 2	− 1	+ 2
95	+ 2	− 4	+ 1	− 5	+ 3	− 2	+ 4	+ 1	+ 2	− 8	− 2	+ 13
96	− 2	+ 1	+ 5	− 4	− 4	+ 3	− 2	+ 4	+ 11	− 11	− 4	+ 2
97	+ 1	− 3	− 2	− 2	− 4	0	0	− 4	+ 7	− 7	0	+ 5
98	+ 3	− 2	+ 3	+ 1	0	+ 4	+ 3	+ 2	+ 12	− 7	− 6	+ 3
99	− 4	+ 2	+ 3	− 5	+ 2	0	+ 2	+ 6	+ 10	− 7	− 3	− 4
100	− 1	+ 1	+ 4	− 2	− 2	0	− 3	− 2	+ 7	+ 5	+ 2	− 4
101	− 4	+ 1	− 1	− 2	0	+ 2	+ 1	+ 1	+ 3	+ 1	+ 7	+ 5
102	+ 2	− 6	− 4	0	− 1	− 1	0	− 3	0	+ 2	0	+ 6
103	− 2	− 2	+ 1	+ 5	− 2	+ 1	+ 2	+ 3	− 1	+ 6	+ 4	+ 5
104	+ 1	− 1	− 6	0	+ 1	0	+ 2	− 2	− 5	+ 6	+ 1	+ 7
105	− 1	− 2	− 7	+ 5	+ 1	0	− 7	+ 3	− 8	+ 7	− 2	+ 10
106	+ 1	− 4	0	+ 4	− 1	+ 5	− 1	− 1	− 6	+ 6	− 4	+ 3
107	− 4	+ 2	+ 1	+ 1	+ 2	− 4	− 3	− 8	− 8	+ 1	− 4	0
108	− 1	+ 3	− 1	0	− 2	− 2	+ 1	0	− 11	+ 3	+ 3	− 6
109	− 1	− 6	+ 5	− 3	− 1	− 1	+ 1	0	− 3	+ 5	0	+ 4
110	+ 4	0	+ 6	− 2	+ 2	+ 3	+ 3	+ 5	− 4	− 2	+ 3	− 1
111	− 1	0	+ 3	− 1	0	− 3	− 1	− 4	− 8	0	+ 9	+ 2
112	− 1	+ 1	+ 6	− 3	− 2	− 1	0	− 1	+ 1	0	0	+ 4
113	+ 3	− 2	+ 5	0	+ 3	− 1	+ 1	− 1	0	− 1	0	+ 7
114	+ 1	0	+ 2	− 7	0	+ 2	+ 4	0	+ 9	+ 2	0	− 1
115	+ 1	+ 2	+ 6	− 6	0	− 1	+ 1	+ 2	+ 15	0	− 3	− 3
116	− 5	+ 2	+ 6	− 3	− 2	0	− 1	− 1	+ 9	+ 5	+ 4	− 3
117	− 3	+ 2	− 1	+ 3	0	+ 2	+ 1	0	+ 7	+ 1	+ 4	− 1
118	+ 1	+ 2	− 2	− 1	+ 2	− 1	+ 1	0	+ 11	− 2	+ 9	+ 1
119	− 2	− 2	0	− 3	+ 1	+ 4	− 1	+ 4	0	+ 12	0	+ 1
120	− 1	+ 3	− 5	+ 2	0	+ 3	+ 3	− 2	− 5	+ 14	+ 6	+ 2
121	+ 4	0	− 4	+ 2	0	− 3	+ 3	− 2	− 7	+ 6	+ 1	− 3
122	− 2	0	− 2	+ 2	− 1	0	− 1	+ 2	− 8	+ 10	− 3	− 1
123	0	− 1	− 3	− 1	+ 4	0	+ 1	0	− 3	+ 9	− 6	+ 1
124	+ 2	− 3	+ 1	+ 1	− 1	− 2	+ 1	+ 1	− 8	+ 9	− 4	− 1
125	+ 2	− 6	− 5	+ 7	0	+ 2	− 4	+ 4	− 13	0	+ 2	− 2
126	+ 3	0	0	+ 6	+ 2	+ 1	+ 5	+ 3	− 11	0	+ 5	− 3
127	+ 3	+ 1	0	+ 3	+ 1	+ 1	− 5	0	− 8	0	+ 5	+ 2
128	− 1	− 1	+ 1	+ 5	− 2	− 1	+ 3	− 1	− 5	− 4	+ 6	+ 1
129	0	+ 3	+ 5	+ 1	+ 2	+ 1	− 1	0	+ 3	− 7	+ 5	+ 3
130	+ 2	− 3	+ 1	0	− 3	− 1	− 1	− 2	+ 1	− 2	0	+ 2
131	0	+ 1	− 1	+ 3	+ 3	− 1	0	+ 4	+ 4	− 5	− 3	0
132	− 1	0	+ 2	− 1	+ 2	+ 1	− 1	− 2	+ 8	− 4	− 3	− 6
133	+ 1	+ 2	+ 2	− 1	− 2	− 3	− 2	+ 2	+ 2	+ 4	+ 5	− 6
Sums +	+42	+35	+84	+67	+39	+46	+50	+51	+122	+133	+90	+105
Sums −	−48	−57	−51	−59	−41	−30	−50	−42	−207	−77	−62	−57
General Sum	− 6	−22	+33	+ 8	− 2	+16	0	+ 9	− 85	+ 56	+28	+ 48

Period.	53^s6.	53^c6.	53^s8.	53^c8.	53^s10.	53^c10.	51^s4.	51^c4.	49^s7.	49^c7.	49^s9.	49^c9.
86	+ 3	+ 4	+ 3	0	0	− 2	0	+ 2	− 1	− 1	− 2	+ 2
87	− 2	+ 2	+ 3	− 4	+ 2	− 3	− 2	− 2	+ 2	0	+ 1	+ 2
88	− 2	0	+ 6	+ 2	+ 2	+ 3	0	− 2	− 4	− 3	− 4	+ 3
89	0	− 4	+ 2	0	− 2	+ 4	+ 1	0	− 3	+ 1	− 1	+ 2
90	− 5	0	+ 1	− 1	− 4	0	+ 3	0	+ 2	− 5	0	+ 1
91	− 6	− 2	+ 1	+ 1	− 2	− 4	+ 5	− 3	− 2	− 2	− 5	− 4
92	− 5	+ 3	+ 4	+ 2	+ 1	+ 2	+ 3	+ 1	+ 5	− 3	+ 3	+ 2
93	− 4	+ 1	0	+ 1	+ 2	− 1	+ 1	+ 1	− 2	− 1	− 2	− 2
94	0	− 1	− 4	− 2	− 2	− 1	+ 1	− 1	− 3	0	+ 2	− 3
95	− 6	− 3	+ 3	− 2	− 2	− 1	+ 4	0	− 2	0	+ 1	+ 3
96	+ 3	− 1	+ 4	− 1	− 2	+ 5	0	+ 3	+ 1	+ 4	− 1	− 2
97	+ 2	− 1	+ 2	− 3	+ 1	+ 2	0	+ 2	0	+ 2	+ 2	− 1
98	+ 3	− 7	+ 1	+ 4	+ 2	− 2	− 7	+ 2	− 2	− 4	+ 4	− 3
99	+ 1	− 4	+ 1	+ 6	+ 1	− 1	− 4	+ 6	− 3	+ 1	− 2	+ 2
100	− 1	− 1	0	− 5	0	+ 2	+ 2	+ 1	+ 1	+ 1	+ 4	+ 1
101	− 2	− 3	+ 5	− 4	− 3	+ 2	− 3	− 3	+ 2	− 5	+ 1	− 1
102	+ 5	+ 1	+ 2	+ 2	+ 2	+ 1	− 1	+ 3	− 2	+ 1	0	− 2
103	+ 2	0	− 5	− 1	− 2	+ 2	− 1	− 4	− 1	− 4	− 4	− 2
104	− 4	− 5	0	− 7	− 3	0	− 1	− 1	+ 4	− 1	− 2	− 1
105	− 2	+ 2	+ 3	+ 3	− 1	+ 1	+ 4	+ 1	0	− 4	− 1	+ 5
106	− 3	+ 3	+ 1	+ 4	− 3	− 2	+ 1	0	− 1	− 3	0	+ 1
107	+ 4	0	− 1	0	+ 2	− 1	− 1	0	− 3	− 2	− 1	0
108	− 1	+ 1	− 1	− 1	+ 5	0	− 2	− 1	− 1	+ 1	0	+ 2
109	− 1	0	+ 1	+ 2	− 2	+ 1	+ 3	+ 5	+ 5	+ 1	+ 2	+ 1
110	− 8	0	− 3	+ 5	− 1	+ 2	− 4	− 1	+ 3	+ 3	− 1	− 1
111	− 3	+ 1	− 4	+ 1	0	− 6	− 5	+ 5	+ 6	+ 1	0	− 1
112	− 5	− 3	− 6	− 1	− 3	− 1	0	+ 3	− 1	+ 5	0	− 1
113	− 1	− 6	+ 2	− 2	− 2	− 1	− 5	− 2	− 4	+ 1	− 1	− 1
114	0	− 2	+ 4	− 1	+ 4	0	− 2	0	− 5	− 1	+ 3	− 1
115	− 2	+ 2	− 2	+ 4	− 6	+ 1	+ 2	− 1	− 3	− 6	+ 1	− 2
116	− 2	− 5	− 2	− 1	+ 2	0	− 1	− 3	0	− 2	+ 2	+ 4
117	− 1	+ 2	+ 1	+ 3	− 1	− 2	− 4	− 1	+ 3	− 2	− 1	0
118	+ 2	− 2	+ 1	− 2	+ 1	− 2	+ 3	+ 2	+ 2	− 1	− 3	0
119	+ 2	− 2	+ 1	+ 7	+ 4	+ 2	+ 1	0	+ 3	− 4	− 2	− 1
120	0	+ 1	− 5	+ 2	0	− 2	+ 2	+ 2	+ 1	− 2	0	− 2
121	0	+ 2	+ 1	− 5	+ 2	− 2	0	− 1	+ 1	− 1	+ 2	− 3
122	− 2	− 2	+ 2	+ 7	0	0	− 4	+ 1	− 2	− 1	− 6	+ 1
123	+ 2	− 3	− 3	+ 3	+ 1	− 2	0	0	0	+ 2	0	− 1
124	0	+ 3	− 1	− 3	+ 6	0	+ 1	− 4	0	− 1	+ 1	− 2
125	− 1	+ 5	+ 2	+ 6	+ 7	0	+ 2	+ 5	+ 5	+ 2	− 6	− 4
126	+ 3	0	+ 3	+ 3	− 2	− 1	− 3	+ 3	− 1	− 2	0	0
127	0	+ 3	− 4	+ 3	− 4	+ 3	− 2	0	− 6	− 3	+ 1	− 4
128	− 2	+ 1	− 2	0	0	− 1	0	− 3	− 1	0	+ 5	− 3
129	0	+ 2	+ 1	− 1	− 2	+ 1	+ 1	+ 1	− 2	+ 2	+ 3	0
130	− 2	− 2	+ 2	+ 1	+ 3	+ 3	− 3	0	+ 1	0	− 2	+ 1
131	+ 3	0	+ 1	0	0	− 5	0	− 3	+ 5	0	+ 1	0
132	+ 1	+ 6	+ 4	+ 1	+ 2	− 2	+ 3	− 3	+ 2	− 1	− 2	0
133	− 6	0	− 1	+ 3	− 2	+ 1	− 1	0	+ 5	+ 1	+ 1	− 3
ns +	+36	+45	+68	+76	+52	+38	+43	+49	+59	+29	+40	+32
ns −	−79	−59	−44	−47	−51	−45	−56	−39	−55	−65	−49	−52
neral um	−43	−14	+24	+29	+ 1	− 7	−13	+10	+ 4	−36	− 9	−20

Period.	$43^{s}5$.	$43^{c}5$.	$41^{s}1$.	$41^{c}1$.	$41^{s}3$.	$41^{c}3$.	$41^{s}6$.	$41^{c}6$.	$39^{s}4$.	$39^{c}4$.	$34^{s}1$.	$34^{c}1$.
86	− 2	0	0	+ 2	− 4	− 4	+ 1	+ 5	− 2	+ 8	− 2	+ 1
87	− 1	− 1	− 3	+ 1	+ 1	+ 1	− 1	0	+ 4	− 2	+ 1	+ 1
88	− 3	− 7	+ 4	− 3	+ 7	− 2	− 1	+ 1	− 1	0	− 6	+ 2
89	− 6	+ 3	+ 1	+ 1	+ 1	− 6	− 4	+ 3	+ 2	+ 3	− 4	− 3
90	+ 3	− 1	− 3	− 2	− 4	+ 1	− 1	+ 1	+ 6	− 3	+ 1	+ 1
91	− 1	− 1	0	0	− 7	+ 6	0	+ 1	− 2	+ 1	+ 4	+ 4
92	0	0	+ 1	+ 2	− 4	+ 5	+ 2	− 1	0	+ 1	+ 1	− 7
93	+ 2	+ 3	+ 3	0	+ 1	+ 2	0	+ 1	+ 5	− 7	+ 3	− 5
94	− 3	− 2	+ 2	− 4	− 3	− 4	− 2	0	− 2	0	+ 2	− 2
95	+ 3	+ 1	− 3	0	+ 2	− 4	− 3	+ 2	− 2	0	+ 1	+ 5
96	− 1	+ 6	− 1	0	+ 3	+ 1	− 3	+ 1	− 6	− 2	+ 2	+ 3
97	+ 3	+ 3	+ 1	− 4	+ 5	+ 1	− 1	0	− 1	0	+ 1	0
98	0	− 1	− 1	− 8	− 2	− 5	+ 1	+ 3	+ 2	+ 2	+ 6	+ 3
99	+ 1	+ 3	− 1	+ 5	−10	− 4	+ 1	+ 1	− 2	+ 3	+ 5	+ 2
100	− 2	+ 3	− 2	− 6	− 6	− 5	− 2	0	0	+ 3	0	+ 1
101	− 7	+ 1	− 1	+ 2	0	− 2	0	0	+ 3	− 2	− 2	+ 3
102	− 5	− 8	0	+ 1	+ 1	+ 1	0	+ 3	+ 5	0	− 5	+ 3
103	+ 1	− 5	− 1	− 4	− 5	+ 2	+ 1	0	− 2	− 3	− 3	+ 3
104	+ 1	− 3	− 1	− 2	− 2	+ 2	+ 2	+ 1	− 1	− 3	+ 2	+ 5
105	+ 2	+ 5	− 3	− 4	+ 4	0	− 3	− 2	− 2	+ 1	− 4	0
106	+ 2	+ 3	− 2	+ 1	+ 2	− 2	− 2	− 1	− 1	+ 2	− 3	+ 1
107	+ 4	− 1	− 3	− 3	+ 5	− 1	+ 1	0	− 9	+ 5	− 2	0
108	+ 1	+ 1	+ 1	+ 3	+ 4	− 3	− 2	+ 1	+ 1	+ 2	− 1	− 1
109	− 1	+ 3	+ 2	− 4	− 4	+ 2	− 1	+ 1	0	+ 3	+ 5	+ 4
110	− 7	+ 1	− 3	+ 1	0	+ 4	+ 2	− 2	+ 4	− 1	+ 1	+ 4
111	0	0	− 3	0	− 3	+ 8	− 4	− 1	− 3	− 2	− 2	− 1
112	− 1	+ 2	− 2	+ 2	+ 4	− 1	− 3	+ 1	0	+ 2	− 6	− 2
113	0	+ 3	− 2	− 2	+ 2	− 1	+ 1	0	− 1	− 3	+ 5	+ 1
114	− 4	− 3	− 1	+ 4	− 3	− 4	+ 1	0	− 6	+ 2	+ 6	− 2
115	+ 2	− 1	− 2	− 3	− 4	+ 3	− 7	− 3	− 3	+ 1	0	0
116	0	− 4	+ 1	− 1	− 1	+ 4	+ 3	+ 7	+ 2	− 1	+ 1	− 1
117	0	+ 4	+ 1	− 1	+ 4	+ 5	+ 1	+ 5	− 4	− 3	+ 1	− 4
118	+ 2	+ 1	0	− 1	− 1	− 5	+ 1	+ 5	− 4	+ 7	+ 2	0
119	0	+ 5	− 4	− 5	0	− 4	− 1	+ 1	+ 2	− 1	+ 2	+ 5
120	+ 2	+ 4	− 1	− 5	+ 5	− 2	+ 3	− 2	+ 4	+ 2	+ 1	+ 3
121	+ 2	− 1	0	0	− 1	− 3	+ 5	0	+ 4	+ 2	− 3	+ 2
122	− 6	− 6	− 7	− 5	− 5	+ 2	0	+ 1	− 4	+ 6	− 1	− 3
123	− 4	0	− 1	+ 1	− 3	+ 5	0	− 3	+ 1	− 2	+ 2	− 2
124	− 4	+ 1	− 5	+ 2	− 1	+ 7	− 1	− 1	− 2	0	+ 2	− 4
125	+ 1	+ 4	− 2	− 3	+ 2	− 1	− 1	− 4	+ 1	− 4	+ 1	+ 2
126	0	− 2	+ 2	+ 4	+ 5	− 8	0	+ 1	− 2	− 2	+ 3	− 1
127	0	+ 2	+ 1	+ 5	− 1	− 1	0	− 6	0	+ 1	− 1	0
128	0	0	− 1	+ 2	0	− 4	− 1	+ 1	− 2	+ 7	+ 4	− 1
129	− 3	− 2	+ 1	+ 2	− 2	0	− 1	0	+ 3	0	0	+ 1
130	+ 1	+ 1	+ 1	0	− 2	+ 1	− 1	− 1	0	+ 1	− 2	− 2
131	0	− 1	0	+ 1	− 2	− 1	− 1	− 2	− 2	− 2	− 1	+ 2
132	+ 4	− 1	− 1	+ 4	− 1	− 1	− 3	0	+ 2	− 2	− 1	− 1
133	+ 1	+ 7	+ 1	+ 1	0	0	0	− 4	− 3	+ 2	+ 8	+ 2
Sums +	+38	+70	+23	+47	+58	+63	+26	+47	+51	+67	+73	+64
Sums −	−61	−51	−60	−70	−81	−78	−50	−33	−69	−45	−49	−42
General Sum	−23	+19	−37	−23	−23	−15	−24	+14	−18	+22	+24	+22

Period.	$34^s2.$	$34^c2.$	$31^s1.$	$31^c1.$	$31^s2.$	$31^c2.$	$31^s3.$	$31^c3.$	$29^s4.$	$29^c4.$	$28^s5.$	$28^c5.$
86	− 2	+ 2	+ 1	− 2	+ 1	− 2	− 1	+ 3	+ 2	+ 5	− 4	0
87	− 2	− 4	− 2	− 1	+ 5	− 5	+ 3	0	0	+ 2	+ 3	− 3
88	0	0	− 2	− 1	− 1	+ 1	0	− 4	− 1	− 1	+ 1	+ 1
89	− 2	+ 2	− 2	− 3	+ 2	0	0	+ 3	− 2	0	− 2	+ 1
90	+ 1	0	− 3	+ 3	0	+ 1	− 5	+ 1	− 6	0	− 1	− 4
91	0	− 2	− 2	+ 2	− 2	+ 3	+ 1	+ 2	+ 3	− 3	0	+ 3
92	+ 2	− 3	+ 2	+ 1	+ 2	− 2	− 4	− 2	0	− 3	0	− 4
93	− 1	0	0	+ 4	− 1	− 2	+ 3	+ 1	+ 2	− 2	0	− 2
94	− 2	+ 1	− 4	+ 5	+ 1	− 2	− 3	− 2	− 2	− 2	+ 2	+ 3
95	− 4	0	− 4	+ 5	− 1	0	− 1	+ 1	+ 3	− 1	0	0
96	+ 2	0	− 1	+ 8	+ 2	− 4	+ 2	− 1	− 1	+ 1	+ 1	− 2
97	+ 1	+ 3	− 9	+ 4	− 1	− 2	− 5	+ 2	+ 2	− 1	+ 2	0
98	− 4	+ 1	− 6	+ 2	+ 1	− 6	+ 2	− 3	− 2	0	+ 1	+ 1
99	− 4	0	− 9	− 1	+ 3	− 2	+ 4	+ 2	− 3	− 3	0	+ 3
100	+ 3	− 1	− 6	− 4	− 1	− 1	0	− 1	− 1	− 2	− 1	+ 1
101	+ 4	− 3	+ 2	− 5	0	0	+ 4	− 1	+ 3	+ 2	+ 3	+ 2
102	− 2	+ 1	+ 2	− 4	0	+ 6	+ 2	+ 1	+ 2	− 3	− 1	+ 2
103	0	+ 1	+ 6	− 1	+ 1	+ 2	+ 4	+ 1	− 2	− 5	− 3	+ 1
104	+ 3	− 2	+ 4	− 3	− 1	− 2	+ 1	− 2	+ 1	− 1	− 1	− 1
105	+ 3	+ 1	− 3	+ 4	+ 4	+ 2	+ 4	+ 2	+ 4	− 2	− 4	− 2
106	0	0	− 4	− 1	+ 1	− 1	0	− 1	0	− 2	0	+ 1
107	+ 2	+ 2	+ 3	− 4	0	− 3	+ 3	0	− 2	− 2	+ 8	+ 2
108	− 2	+ 1	+ 1	− 5	+ 1	− 4	0	0	− 2	0	+ 1	+ 1
109	+ 1	− 1	+ 5	0	− 3	0	+ 1	− 3	− 1	0	0	+ 7
110	+ 4	+ 4	+ 1	0	− 1	− 6	0	+ 2	+ 1	− 1	+ 4	0
111	0	0	+ 8	+ 7	+ 1	+ 3	+ 2	0	0	+ 1	+ 1	− 1
112	+ 2	− 4	− 2	+ 4	+ 6	+ 2	− 4	− 2	0	+ 3	− 3	− 3
113	0	+ 2	− 2	0	− 2	0	0	+ 1	− 1	+ 1	− 2	0
114	+ 3	− 1	+ 3	+ 1	− 6	− 1	+ 1	+ 2	+ 4	− 4	+ 1	0
115	+ 2	+ 2	− 2	0	− 6	− 3	+ 1	+ 2	+ 1	0	+ 2	0
116	− 6	− 2	− 6	− 5	− 1	+ 2	− 1	+ 1	− 1	+ 8	+ 1	+ 1
117	0	0	− 3	+ 2	− 3	+ 5	+ 1	0	+ 1	− 1	+ 3	+ 2
118	0	+ 2	+ 2	− 2	+ 2	0	− 3	+ 1	+ 3	− 1	− 3	0
119	0	− 7	+ 2	+ 2	0	− 2	− 2	− 1	+ 2	− 1	− 2	+ 3
120	+ 2	+ 6	+ 1	0	− 4	− 3	+ 4	0	+ 5	+ 1	+ 3	− 2
121	+ 2	0	0	+ 5	− 1	− 1	+ 1	+ 2	0	− 1	+ 2	+ 1
122	− 2	0	− 7	− 2	+ 2	− 4	+ 2	− 2	0	+ 3	+ 3	− 1
123	− 2	0	− 7	− 3	+ 3	+ 2	− 4	+ 2	− 3	+ 2	0	+ 1
124	0	− 2	− 10	− 1	+ 4	+ 4	+ 3	+ 1	0	+ 2	0	− 1
125	+ 2	+ 8	− 4	− 4	0	0	+ 3	+ 5	+ 1	− 2	+ 2	− 2
126	0	0	− 1	− 10	+ 1	− 2	+ 2	− 4	+ 1	− 2	+ 5	+ 3
127	+ 1	− 1	+ 6	− 6	+ 1	+ 2	− 2	− 3	− 1	0	+ 2	− 2
128	− 2	0	+ 3	− 4	− 2	− 3	− 3	− 2	+ 2	0	+ 2	0
129	0	+ 2	+ 2	+ 3	− 2	0	0	− 6	− 1	+ 1	0	0
130	− 2	− 2	+ 4	+ 3	+ 1	0	+ 3	+ 1	− 2	+ 2	0	0
131	+ 2	0	+ 5	+ 4	− 1	− 1	+ 4	− 1	+ 4	+ 2	0	+ 4
132	+ 3	+ 3	− 6	+ 7	− 1	0	+ 1	0	− 1	− 1	+ 1	− 2
133	− 1	+ 3	− 6	+ 1	+ 4	+ 2	− 2	− 2	0	− 2	+ 2	− 2
Sums +	+45	+47	+ 63	+77	+49	+37	+62	+39	+47	+36	+56	+44
Sums −	−40	−35	−113	−72	−41	−64	−40	−43	−35	−49	−27	−34
General Sum	+ 5	+12	− 50	+ 5	+ 8	−27	+22	− 4	+12	−13	+29	+10

Period.	27^s3.	27^c3.	27^s4.	27^c4.	27^s6.	27^c6.	23^s1.	23^c1.	23^s4.	23^c4.	22^s1.	22^c1.
86	0	+ 3	0	− 5	− 5	− 2	+ 3	+ 3	+ 1	0	− 2	+ 3
87	+ 2	− 9	+ 2	+ 1	+ 4	− 3	− 5	− 1	+ 4	− 1	+ 5	+ 3
88	− 3	− 5	− 4	+ 2	+ 3	− 3	+ 4	+ 2	− 4	− 5	+ 3	+ 1
89	− 2	+ 1	− 1	− 2	+ 2	− 3	+ 4	0	0	+ 2	− 1	− 1
90	0	+ 4	− 2	+ 1	+ 3	+ 1	+ 3	− 6	− 3	+ 1	+ 2	0
91	− 2	+ 1	+ 1	+ 5	+ 5	+ 2	− 1	− 2	+ 1	− 5	− 1	0
92	− 2	+ 5	− 3	+ 2	− 1	− 3	− 5	+ 1	− 1	+ 2	0	− 2
93	+ 4	+ 1	− 2	+ 3	− 2	− 3	− 4	− 1	+ 2	− 1	+ 3	− 3
94	− 1	− 1	+ 1	− 7	+ 1	− 2	0	0	− 1	− 1	− 1	− 1
95	− 1	− 6	− 3	− 3	+ 1	+ 1	− 4	+ 3	+ 2	− 2	− 2	− 1
96	− 6	− 4	+ 1	+ 3	− 2	− 5	− 4	+ 1	+ 1	0	− 2	− 1
97	− 4	+ 2	0	+ 3	+ 1	+ 2	− 2	+ 1	+ 3	− 2	+ 2	+ 3
98	0	+ 4	− 2	− 6	+ 6	0	+ 6	0	− 3	0	− 1	− 1
99	0	+ 3	+ 5	− 1	+ 1	0	− 3	− 5	+ 1	− 2	+ 1	+ 2
100	0	+ 1	+ 3	− 1	+ 1	− 2	− 2	+ 3	− 1	+ 4	− 2	− 4
101	− 1	+ 4	+ 1	− 1	+ 1	− 2	− 2	− 2	− 2	0	− 3	+ 1
102	0	+ 1	0	+ 5	+ 1	− 1	− 3	− 3	+ 3	− 2	0	− 1
103	+ 1	− 4	0	+ 1	+ 3	0	+ 3	− 2	0	+ 1	− 1	0
104	− 1	− 2	+ 4	+ 5	+ 1	+ 2	+ 1	+ 2	− 3	+ 2	+ 1	0
105	− 4	+ 2	+ 1	+ 5	− 1	0	0	+ 4	0	− 6	+ 2	+ 3
106	− 2	+ 5	+ 1	− 3	0	+ 2	+ 2	− 2	− 1	+ 1	− 2	+ 2
107	− 3	− 1	− 4	0	+ 4	+ 5	+ 2	+ 1	+ 2	− 1	− 3	+ 4
108	− 3	+ 2	+ 2	− 1	+ 3	− 2	− 6	− 4	− 3	− 1	+ 2	+ 2
109	0	− 1	+ 3	− 1	0	+ 1	+ 5	+ 3	+ 7	+ 6	+ 1	0
110	+ 5	− 2	0	+ 6	+ 1	+ 2	+ 3	+ 2	− 2	+ 2	0	+ 1
111	+ 3	− 5	+ 1	− 1	+ 2	+ 3	+ 5	+ 1	− 1	+ 1	0	+ 1
112	+ 1	− 3	− 2	+ 1	− 2	− 1	+ 2	− 3	− 3	0	+ 1	+ 1
113	− 5	0	− 4	+ 1	+ 1	+ 2	0	+ 1	− 2	0	− 2	0
114	− 4	+ 1	− 2	0	− 2	+ 4	0	− 1	0	+ 3	− 1	+ 3
115	− 4	− 3	0	− 6	+ 3	+ 2	+ 1	+ 3	+ 1	0	− 1	− 2
116	− 10	− 4	− 4	− 2	+ 1	+ 2	− 4	+ 5	+ 3	+ 1	+ 7	0
117	− 2	− 3	− 1	+ 1	− 2	0	+ 1	+ 2	+ 2	− 2	− 3	+ 1
118	+ 1	+ 3	+ 1	0	− 1	− 2	+ 1	+ 2	+ 2	− 2	+ 2	0
119	+ 1	+ 4	+ 1	+ 1	− 5	− 1	+ 3	+ 3	− 5	− 3	+ 3	+ 5
120	0	− 4	− 2	− 3	+ 4	− 2	+ 2	+ 2	− 4	0	− 1	− 1
121	− 1	+ 2	+ 2	0	− 3	+ 4	− 2	+ 3	+ 3	+ 3	+ 1	+ 1
122	+ 3	− 1	− 4	− 3	− 4	0	− 7	+ 1	+ 6	− 10	− 5	− 1
123	− 3	− 4	+ 7	+ 4	+ 1	0	0	− 3	− 3	+ 2	+ 1	+ 1
124	− 2	+ 3	− 2	+ 1	0	+ 1	+ 2	0	+ 3	+ 3	0	0
125	− 1	+ 1	− 6	− 3	+ 2	− 1	− 9	+ 5	0	+ 2	+ 3	+ 1
126	+ 2	+ 3	0	− 2	0	0	+ 1	− 3	+ 2	− 4	0	− 4
127	+ 3	+ 3	+ 3	0	− 1	− 3	+ 3	− 1	0	− 3	0	0
128	0	− 6	− 3	+ 2	0	− 2	+ 3	+ 3	− 3	+ 4	+ 4	+ 3
129	− 2	− 3	+ 2	− 2	+ 1	− 2	− 2	+ 1	+ 2	− 3	+ 1	0
130	0	0	+ 1	+ 1	− 1	− 1	+ 2	+ 2	0	+ 2	+ 2	− 4
131	0	− 1	+ 2	+ 1	0	− 1	+ 1	+ 2	0	+ 2	− 2	+ 2
132	− 2	+ 1	− 1	− 1	+ 3	− 2	− 2	0	− 2	+ 2	0	+ 1
133	− 1	+ 3	+ 2	+ 2	+ 1	− 5	− 3	+ 2	− 3	− 1	− 4	− 1
Sums +	+ 26	+ 63	+ 47	+ 57	+ 61	+ 36	+ 63	+ 64	+ 51	+ 46	+ 47	+ 45
Sums −	− 72	− 72	− 52	− 54	− 32	− 54	− 70	− 39	− 50	− 57	− 40	− 28
General Sum	− 46	− 9	− 5	+ 3	+ 29	− 18	− 7	+ 25	+ 1	− 11	+ 7	+ 17

Period.	sin D.	cos. D.	sin 2D.	cos 2D.	sin 3D.	cos 3D.	sin 4D.	cos 4D.	sin 5D.	cos 5D.	μ.	No. of Obs.
86	+ 3	− 8	+ 1	− 2	− 1	0	+ 1	0	0	+ 1	− 1	118
87	+ 2	− 2	−10	0	0	0	+ 7	+ 2	− 2	− 2	− 13	134
88	+ 9	− 16	+ 1	− 4	− 4	+ 2	0	+ 1	+ 1	− 3	− 7	123
89	0	− 3	0	+ 3	0	− 5	0	+ 4	+ 2	+ 1	0	121
90	+ 2	− 6	− 1	+ 1	− 4	+ 2	+ 5	− 6	+ 1	+ 3	− 6	126
91	+ 5	+ 8	0	+ 6	− 1	− 1	− 1	− 1	+ 2	+ 1	0	130
92	− 8	− 2	+ 3	+ 1	0	+ 1	− 1	− 4	+ 1	0	+ 2	128
93	+ 13	0	+ 4	+ 5	− 8	− 3	+ 5	+ 2	− 1	− 2	− 9	109
94	− 4	− 1	− 4	− 1	0	− 2	+ 5	+ 4	− 2	− 3	− 2	109
95	− 4	+ 6	− 1	− 1	0	+ 2	+ 5	− 1	− 2	0	− 4	138
96	− 6	− 19	− 3	−15	− 1	+ 7	+ 3	0	− 3	− 3	− 5	140
97	+ 9	+ 8	+ 1	+ 2	− 4	− 2	+ 5	− 1	− 2	+ 3	− 16	135
98	− 5	+ 11	0	+ 8	− 3	− 2	+ 4	− 3	− 1	+ 2	− 2	119
99	− 6	− 5	+ 2	− 1	− 1	+ 2	− 1	− 3	+ 5	− 1	+ 5	111
100	+ 1	− 2	− 4	+ 1	+ 3	0	− 3	− 2	+ 4	− 2	+ 1	123
101	− 6	− 1	+ 1	− 1	− 1	+ 3	+ 1	− 2	+ 3	− 1	+ 4	125
102	+ 1	+ 3	− 4	− 5	+ 1	+ 5	0	+ 1	+ 1	− 3	+ 1	124
103	+ 1	+ 10	− 2	+ 6	− 1	− 3	+ 5	+ 2	0	− 1	+ 2	108
104	− 12	+ 4	+ 3	+ 3	+ 1	− 1	− 2	− 2	+ 2	+ 4	+ 6	128
105	− 4	− 5	+ 5	− 1	− 4	+ 2	0	− 2	+ 4	− 2	+ 5	116
106	+ 9	+ 6	0	+ 2	− 2	− 2	0	0	+ 2	− 1	− 5	121
107	+ 3	+ 8	+ 1	+ 2	− 2	− 1	0	+ 1	+ 1	0	− 1	116
108	+ 1	+ 6	+ 5	+ 2	+ 2	0	− 3	− 4	0	+ 4	+ 5	123
109	+ 10	0	+ 3	0	− 5	+ 2	− 2	− 4	+ 7	+ 4	+ 2	114
110	− 11	− 3	+ 1	− 2	+ 5	− 2	− 5	+ 4	+ 1	− 2	+ 11	125
111	− 1	− 2	− 3	+ 1	+ 2	+ 1	+ 1	− 4	− 2	+ 4	− 3	96
112	− 6	0	+ 1	+ 5	+ 2	− 4	− 2	+ 2	0	+ 1	+ 4	104
113	+ 5	+ 10	− 4	+ 6	0	− 3	+ 2	0	0	+ 2	− 4	115
114	+ 3	− 13	− 4	− 1	+ 2	+ 3	0	− 7	− 1	+ 3	− 2	93
115	− 7	− 4	+ 5	+ 4	+ 1	− 2	− 3	0	+ 1	− 2	+ 10	115
116	− 5	− 22	− 2	− 5	0	+ 4	0	− 4	+ 3	+ 2	+ 6	115
117	− 7	− 4	− 1	+ 2	+ 3	0	0	− 2	− 2	− 1	+ 8	113
118	0	− 2	+ 1	0	0	0	0	+ 3	0	− 5	+ 4	117
119	+ 5	+ 4	− 6	− 2	− 1	0	+ 4	− 1	+ 1	+ 2	− 5	114
120	+ 2	− 13	− 5	− 7	0	+ 6	0	− 3	+ 2	− 3	− 3	100
121	+ 4	+ 4	+ 4	+ 3	− 1	0	− 3	− 2	+ 2	− 1	+ 2	120
122	+ 6	− 6	+ 3	− 2	− 2	0	− 1	0	+ 4	− 3	+ 5	84
123	+ 6	− 2	− 1	0	− 2	− 1	+ 1	+ 3	+ 3	− 3	− 1	91
124	+ 5	− 3	0	+ 3	0	− 2	− 2	+ 2	+ 2	0	0	114
125	+ 1	− 3	− 8	+ 3	− 1	− 3	+ 7	+ 4	0	+ 2	− 3	90
126	− 3	− 1	− 2	− 1	− 2	0	+ 5	+ 2	− 2	− 2	− 3	134
127	− 4	+ 3	0	+ 6	0	− 3	− 1	+ 1	+ 3	− 1	+ 4	120
128	− 6	− 4	+ 1	− 3	− 2	+ 4	+ 2	− 3	0	0	+ 4	127
129	+ 7	− 5	0	− 1	− 5	0	+ 3	− 1	+ 1	+ 3	− 9	128
130	− 5	− 12	0	− 5	− 1	+ 3	0	− 1	+ 1	− 1	0	112
131	+ 4	− 5	− 1	− 1	− 3	− 1	+ 4	0	− 3	+ 2	− 8	123
132	− 3	− 2	− 1	+ 1	0	0	0	+ 1	+ 1	− 1	+ 1	133
133	− 5	+ 17	+ 4	+10	− 1	− 2	− 1	− 1	0	− 3	+ 4	124
Sums +	+117	+108	+50	+86	+22	+49	+75	+39	+61	+44	+ 96	
Sums −	−118	−176	−67	−61	−63	−45	−31	−64	−23	−52	−112	
General Sum	− 1	− 68	−17	+25	−41	+ 4	+44	−25	+38	− 8	− 16	5646

TABLE IX.

Mean Values for Periods 1 to 89 of the Errors of the Moon's Longitude multiplied by Certain Factors in Tenths of a Second of Arc.

Period.	$80^{s}3$.	$80^{c}3$.	$53^{s}2$.	$53^{c}2$.	$53^{s}4$.	$53^{c}4$.	$53^{s}6$.	$53^{c}6$.	$53^{s}8$.	$53^{c}8$.
1	+ 10	+ 10	+ 14	+ 5	0	− 10	+ 2	− 7	− 6	− 1
2	− 8	+ 16	+ 11	+ 14	− 9	− 3	− 6	+ 9	− 8	− 10
3	− 15	+ 18	+ 18	+ 11	+ 8	+ 1	+ 6	+ 6	0	− 8
4	− 6	− 3	+ 2	+ 7	− 5	+ 6	+ 2	+ 3	+ 7	− 4
5	− 4	− 6	0	+ 4	+ 12	− 4	− 8	+ 1	− 5	+ 5
6	+ 2	− 16	− 1	+ 15	− 1	+ 4	+ 2	0	− 1	+ 5
7	+ 6	− 18	+ 7	+ 19	− 13	+ 9	+ 3	+ 5	− 2	+ 2
8	0	− 16	+ 15	+ 5	− 2	+ 3	+ 2	+ 5	+ 7	+ 2
9	− 4	− 11	+ 6	− 5	− 9	+ 8	− 9	+ 5	− 2	+ 10
10	− 7	+ 3	− 6	− 2	− 4	+ 6	+ 1	+ 7	+ 2	+ 9
11	− 15	− 20	− 16	− 17	+ 10	− 6	− 12	− 9	+ 3	− 13
12	+ 10	− 9	+ 4	− 15	+ 13	+ 10	0	+ 9	− 1	− 21
13	+ 5	− 20	− 15	− 12	+ 2	+ 11	+ 4	+ 7	+ 11	+ 8
14	+ 8	− 9	− 9	− 10	+ 4	+ 14	+ 4	0	+ 3	+ 1
15	+ 7	− 1	− 5	− 4	− 7	+ 17	0	− 1	− 1	+ 1
16	− 6	+ 7	+ 7	− 7	0	+ 5	+ 2	− 18	+ 2	− 2
17	− 7	− 4	+ 8	+ 3	− 4	+ 4	+ 10	− 6	+ 9	0
18	+ 2	− 1	+ 1	+ 3	− 3	− 3	− 1	− 5	0	+ 1
19	− 4	− 11	+ 10	− 6	+ 7	+ 2	− 8	+ 8	− 11	+ 6
20	− 13	− 2	− 5	− 11	+ 14	+ 9	− 5	+ 6	+ 5	− 6
21	− 1	− 6	0	− 7	− 2	− 3	+ 4	− 5	+ 6	− 4
22	+ 8	+ 8	+ 10	+ 6	+ 4	+ 11	− 2	− 5	+ 7	0
23	+ 12	+ 14	+ 21	− 2	− 1	+ 13	+ 2	− 3	− 11	− 6
24	+ 7	+ 19	+ 22	− 5	+ 4	+ 2	− 4	+ 9	+ 3	− 1
25	− 2	+ 20	+ 19	− 9	+ 4	− 1	− 6	+ 4	+ 11	+ 4
26	− 10	+ 4	+ 13	+ 1	+ 7	+ 10	− 3	+ 5	− 2	+ 5
27	+ 8	− 17	− 4	+ 19	+ 2	+ 1	0	− 1	− 3	− 1
28	+ 2	+ 2	− 1	− 4	+ 14	+ 4	0	+ 4	+ 1	− 3
29	+ 6	− 5	+ 5	+ 5	+ 1	+ 1	− 1	− 3	+ 2	0
30	+ 4	− 4	+ 4	0	+ 6	+ 7	0	− 5	− 1	+ 2
31	+ 14	0	+ 9	+ 5	+ 1	+ 9	− 2	− 1	− 3	+ 2
32	− 2	+ 15	− 4	+ 15	+ 1	+ 3	− 4	− 2	− 6	0
33	+ 18	+ 2	+ 13	− 5	+ 9	− 1	− 5	− 4	+ 5	+ 4
34	− 11	− 9	− 16	+ 8	− 7	+ 14	+ 9	− 2	+ 9	− 9
35	− 1	+ 2	0	+ 2	− 3	− 6	+ 1	− 8	− 3	+ 6
36	+ 9	− 1	− 7	− 4	+ 4	− 3	− 4	+ 12	− 2	− 2
37	+ 3	+ 7	0	− 6	− 6	+ 1	− 7	+ 10	+ 3	− 6
38	+ 12	+ 2	− 10	+ 4	− 1	0	− 2	− 7	+ 11	+ 3
39	+ 1	0	+ 1	+ 2	− 9	+ 1	− 4	+ 3	+ 7	+ 6
40	− 14	+ 9	− 9	− 15	+ 1	− 9	+ 5	0	− 8	+ 2
41	− 30	+ 1	− 15	− 28	+ 7	− 1	+ 8	0	− 1	− 6
42	− 17	− 1	− 14	− 11	+ 8	+ 8	+ 10	0	− 1	0
43	+ 1	− 10	− 1	− 13	+ 4	− 10	− 17	+ 1	0	− 9
44	0	− 9	− 8	− 10	− 4	+ 3	− 11	+ 7	− 3	+ 7
45	+ 15	− 3	− 1	− 16	− 13	− 1	+ 10	+ 5	− 1	+ 1
46	+ 12	+ 11	+ 4	− 16	+ 2	− 2	− 7	+ 5	+ 4	+ 3
47	− 4	+ 33	+ 18	− 25	− 3	− 5	− 3	+ 1	+ 11	− 2
48	− 22	+ 13	+ 19	− 10	+ 2	− 4	− 4	+ 1	+ 2	− 1
Sums +	+ 182	+ 216	+ 261	+ 153	+ 151	+ 197	+ 87	+ 138	+ 131	+ 95
Sums −	− 203	− 212	− 147	− 275	− 106	− 72	− 135	− 92	− 82	− 115
General Sum	− 21	+ 4	+ 114	− 122	+ 45	+ 125	− 48	+ 46	+ 49	− 20

Period.	53^s10.	53^c10.	sin g.	cos g.
1	+ 4	+ 7	− 2	+ 15
2	− 12	− 5	− 16	+ 8
3	+ 2	− 5	− 20	+ 8
4	+ 1	+ 4	− 5	− 5
5	+ 10	+ 3	0	− 4
6	− 2	+ 2	+ 7	− 13
7	− 6	− 9	+ 8	− 18
8	− 3	+ 2	0	− 16
9	− 1	+ 1	− 4	− 6
10	0	− 6	− 5	+ 4
11	+ 4	− 15	− 23	− 4
12	− 2	+ 4	+ 1	− 16
13	+ 4	− 4	− 11	− 16
14	− 2	+ 1	0	− 14
15	− 2	0	+ 2	− 6
16	− 1	+ 5	+ 7	+ 7
17	− 5	+ 1	− 6	+ 6
18	+ 3	− 3	− 3	− 2
19	+ 1	0	− 8	+ 9
20	− 11	− 4	+ 3	+ 12
21	− 3	− 8	− 4	+ 6
22	+ 7	− 2	0	− 12
23	+ 5	− 1	− 4	− 21
24	0	+ 7	+ 2	− 21
25	0	+ 11	+ 6	− 21
26	+ 3	+ 5	+ 12	− 4
27	+ 6	0	− 6	+ 19
28	+ 2	0	+ 1	− 4
29	− 1	− 2	− 1	+ 7
30	+ 9	+ 1	+ 1	+ 4
31	+ 4	− 2	− 7	+ 8
32	− 6	0	− 10	− 12
33	− 4	− 3	− 9	+ 11
34	+ 10	− 1	+ 15	− 10
35	+ 3	− 1	+ 1	− 2
36	− 1	+ 4	+ 2	+ 8
37	− 7	+ 2	− 5	+ 2
38	0	− 6	+ 3	+ 10
39	− 6	+ 7	+ 2	0
40	+ 3	+ 4	− 16	− 7
41	− 2	+ 5	− 21	− 25
42	− 8	− 3	− 11	− 15
43	− 4	+ 7	+ 7	− 11
44	− 4	+ 4	+ 6	− 11
45	+ 4	0	+ 16	− 1
46	+ 3	0	+ 13	+ 10
47	0	+ 1	+ 3	+ 32
48	− 3	− 6	− 16	+ 16
ms +	+ 88	+ 88	+ 118	+ 202
ms −	− 96	− 86	− 213	− 297
eneral Sum	− 8	+ 2	− 95	− 95

Period.	80^s3.	80^c3.	53^s2.	53^c2.
49	− 14	− 4	+ 13	0
50	− 8	− 3	+ 6	− 3
51	+ 5	0	+ 5	+ 7
52	+ 10	+ 9	+ 3	+ 16
53	− 7	+ 26	− 9	+ 22
54	− 9	+ 2	− 8	+ 4
55	− 7	− 12	− 14	0
56	+ 3	− 7	− 8	− 3
57	+ 18	− 17	− 25	− 5
58	+ 4	− 3	− 5	+ 1
59	+ 4	+ 2	− 6	− 1
60	− 5	+ 13	− 7	− 11
61	− 21	− 12	+ 14	− 20
62	− 6	− 12	+ 9	− 12
63	− 3	− 24	+ 9	− 24
64	+ 4	− 32	+ 4	− 35
65	+ 18	− 20	+ 1	− 26
66	+ 7	− 20	− 15	− 14
67	+ 12	− 7	− 11	− 8
68	− 2	− 14	− 8	+ 13
69	+ 1	− 21	− 3	+ 21
70	+ 7	− 16	− 1	+ 19
71	+ 23	− 16	+ 4	+ 29
72	+ 21	+ 3	− 1	+ 22
73	+ 8	+ 25	− 14	+ 21
74	+ 1	+ 5	0	+ 4
75	− 4	− 11	− 7	− 8
76	+ 6	− 27	− 16	− 22
77	+ 19	− 9	− 8	− 21
78	+ 20	+ 1	− 9	− 18
79	+ 27	+ 18	− 17	− 26
80	+ 14	+ 23	− 14	− 20
81	+ 16	+ 33	− 34	− 14
82	+ 7	+ 34	− 34	− 3
83	0	+ 19	− 17	+ 4
84	+ 7	+ 20	− 5	+ 19
85	+ 5	+ 26	+ 2	+ 24
86	− 1	+ 13	+ 7	+ 11
87	− 8	+ 12	+ 6	+ 12
88	− 14	+ 1	+ 2	+ 14
89	− 8	+ 1	+ 5	+ 4
	+ 267	+ 286	+ 90	+ 267
	− 117	− 287	− 296	− 294
	+ 150	− 1	− 206	− 27

Period.	53^s4.	53^c4.	53^s6.	53^c6.	53^s8.	53^c8.	53^s10.	53^c10.	sin g.	cos g.
49	+ 8	− 3	+ 5	− 3	− 4	+ 4	− 6	− 3	− 13	− 11
50	+ 5	+ 3	− 3	− 3	− 2	− 7	− 4	0	− 6	− 11
51	− 2	− 6	+ 7	0	− 1	+ 6	+ 3	− 7	+ 3	− 7
52	+ 14	− 5	− 1	+ 9	+ 7	+ 1	− 6	0	+ 16	− 5
53	− 15	+ 7	− 4	+ 2	+ 4	− 6	+ 9	− 4	+ 19	+ 16
54	+ 1	− 2	− 7	0	+ 2	− 4	+ 7	− 2	− 2	+ 9
55	− 1	− 1	+ 2	+ 4	+ 5	+ 4	− 3	+ 6	− 13	+ 6
56	− 13	− 7	− 6	− 11	+ 5	− 6	+ 4	− 4	− 8	− 3
57	− 3	− 10	+ 7	− 7	− 6	− 8	− 6	0	− 20	− 14
58	+ 13	− 6	+ 3	+ 1	+ 9	+ 1	+ 2	+ 1	− 4	− 3
59	− 2	− 10	− 2	− 3	− 2	− 1	+ 9	+ 5	0	− 6
60	+ 5	+ 2	+ 1	+ 3	+ 4	+ 1	+ 3	+ 7	+ 12	− 5
61	− 3	+ 2	+ 7	+ 7	+ 1	0	+ 1	+ 1	+ 8	+ 23
62	− 4	− 1	+ 9	+ 10	− 5	− 7	− 7	+ 6	− 2	+ 15
63	− 1	− 5	+ 3	+ 3	+ 2	− 3	− 1	− 4	− 7	+ 25
64	− 2	− 7	− 3	+ 10	− 4	− 5	− 5	0	− 14	+ 30
65	0	− 8	− 2	+ 6	− 6	+ 10	− 5	− 1	− 19	+ 17
66	− 12	0	− 10	− 2	− 4	− 5	+ 1	+ 5	− 8	+ 18
67	+ 6	− 11	+ 3	− 4	− 1	− 3	+ 1	+ 2	− 9	+ 10
68	− 4	+ 4	+ 4	0	+ 2	0	0	− 8	+ 8	+ 14
69	+ 1	+ 11	− 2	0	0	− 1	− 6	− 4	+ 10	+ 19
70	0	0	0	+ 6	− 7	+ 1	+ 4	− 6	+ 5	+ 17
71	− 5	− 3	− 6	0	+ 1	− 2	+ 4	− 2	− 2	+ 30
72	− 8	+ 2	− 3	− 2	0	+ 1	− 4	+ 7	− 14	+ 17
73	− 2	− 17	− 6	+ 4	+ 3	+ 2	− 3	+ 7	− 24	− 4
74	− 9	− 2	+ 2	− 5	− 3	0	+ 12	+ 3	− 4	0
75	+ 3	− 4	+ 11	+ 13	− 7	− 4	− 1	− 3	+ 10	− 3
76	+ 3	− 4	− 11	+ 8	− 2	0	+ 3	− 3	+ 26	+ 4
77	+ 3	+ 6	− 1	− 3	− 1	0	0	+ 2	+ 14	+ 19
78	0	− 1	+ 7	− 1	0	− 1	− 4	− 2	+ 7	+ 19
79	− 1	+ 1	− 2	− 1	− 2	+ 8	− 3	+ 4	+ 1	+ 29
80	− 12	− 4	+ 7	− 1	− 2	+ 4	− 4	− 6	− 9	+ 21
81	− 6	− 7	+ 16	− 8	+ 2	− 2	− 2	− 3	− 11	+ 35
82	− 20	− 7	− 4	+ 4	+ 3	− 2	+ 1	+ 12	− 13	+ 33
83	− 7	− 7	− 3	+ 6	0	+ 1	+ 1	+ 4	− 9	+ 16
84	+ 9	+ 7	− 3	− 4	+ 2	− 7	− 4	+ 2	+ 3	+ 19
85	+ 2	+ 5	+ 4	+ 1	+ 5	− 2	+ 2	+ 3	+ 2	+ 24
86	− 1	+ 3	+ 3	+ 4	+ 6	− 1	+ 2	− 2	+ 2	+ 13
87	− 4	+ 7	− 5	0	+ 1	− 1	+ 4	− 5	− 6	+ 12
88	− 7	− 2	− 1	+ 5	+ 4	+ 4	0	+ 2	− 13	+ 6
89	− 8	+ 1	− 1	+ 5	− 1	0	+ 3	+ 6	− 5	+ 4
Sums +	+ 73	+ 61	+ 101	+ 111	+ 68	+ 48	+ 76	+ 85	+ 146	+ 520
Sums −	− 152	− 140	− 86	− 58	− 60	− 78	− 74	− 69	− 235	− 52
General Sum	− 79	− 79	+ 15	+ 53	+ 8	− 30	+ 2	+ 16	− 89	+ 468

It is clearly impossible to give the individual observations. Table VIII. (1847 to 1901) and Table IX. (1750 to 1851), however, give for each period of analysis the mean for each period of the product of each error by certain factors. In the column headed ${}_m s_n$ the factor is $2 \sin {}_m A_n$; similarly for ${}_m c_n$ the factor is $2 \cos {}_m A_n$. Columns headed sin D, cos D, sin 2D, cos 2D and μ, are copied from lxiv. p. 580, and the method of their calculation is there explained. Columns headed sin g, cos g, are found in this way. From Table II., No. 23, the values of $g - {}_{53}A_2$ may be calculated for the middle of each period. Let $(g - {}_{53}A_2)_0$ denote the value of $g - {}_{53}A_2$ at the middle of a period; then

$$(g - {}_{53}A_2)_0 = (g - {}_{53}A_2) - k\theta$$

when 2θ is given in the last column of Table II. (in this instance $\theta = -12°4$), and k moves uniformly through a period of analysis from -1 to $+1$.

Now the columns headed sin g, cos g (Table IX.), are derived from those headed ${}_{53}s_2$, ${}_{53}c_2$, thus:

$$(\sin g) = ({}_{53}c_2) \sin (g - {}_{53}A_2)_0 + ({}_{53}s_2) \cos (g - {}_{53}A_2)_0$$
$$(\cos g) = ({}_{53}c_2) \cos (g - {}_{53}A_2)_0 - ({}_{53}s_2) \sin (g - {}_{53}A_2)_0$$

Referring to the definitions of the columns headed ${}_{53}s_2$ and ${}_{53}c_2$, it will be seen that the definition of the columns headed sin g and cos g are "mean values of the products of the errors by $2 \sin (g - k\theta)$ and $2 \cos (g - k\theta)$ respectively."

Now if θ were zero, and if we take means for a very large number of periods, then the means so arrived at are the coefficients of sin g and cos g in the errors, affected (i.) by purely accidental errors of observation, (ii.) by systematic errors (α) due to terms differing very slightly from g in mean motion, $g + \phi$ say, where ϕ is small. Systematic errors from this cause disappear if means are taken over a complete revolution of ϕ, and for solar terms the smallest value of ϕ is $\omega - \omega'$ whose period is 9 years; consequently in 50 years there is no systematic effect from any solar term (β) due to terms $g \pm$ D. This effect is due to the distribution of observations about full moon, and can be approximately allowed for by remarking that the mean value of cos D is $-\frac{1}{2}$ (γ) due to terms $g \pm \text{D} + \phi$, a combination of causes (α) and (β) disappearing in process of time like (α).

Lastly, since θ is not zero, the results are diminished by the factor "mean value of cos $k\theta$," $o \frac{\sin \theta}{\theta}$. Conversely, in forming an estimate of the error of the tabular term, the results arrived at must be increased by the factor $\frac{\theta}{\sin \theta}$.

Coefficients of sin g and cos g derived with the help of the auxiliary angle ${}_{53}A_2$ have, by an oversight, not been printed in Table VIII. They have been compared with the last two

columns on lxiv. p. 415, which have been derived with the help of the auxiliary angle ${}_{80}A_3$. There is an error of sign on that page for sin g, period 98 ; otherwise there is a close agreement between the two sets of figures. As the arithmetic of the two analyses is absolutely different, the check afforded by the comparison is a very thorough one.

As at the present time theory is defective, as regards long-period terms, it is probable that more theoretical short-period terms will from time to time be discovered. Table VIII. will in all probability suffice to compare such terms with observation. Meanwhile Table VIII. is evidence that there are not any unknown short-period terms with coefficients approaching to $0''.5$ and with arguments at all approximating to any of the auxiliary angles, with the possible exception of ${}_{53}A_2$, the angle associated with the mean anomaly.

In Table X. I complete my analysis for the terms that I have selected. The first column contains the reference number. The second column contains the argument associated with that reference number in Table II., III., or IV. In the case of solar terms the argument is written in a second form, suggested by the consideration of systematic errors (β) mentioned above. The order of terms is arranged thus : first the solar terms were put into groups, the members of the same group differing by multiples of D only, and the groups being arranged according to the part of the second form which is independent of D, the smallest mean motions being put first ; next the planetary terms were put in ; a few whose mean motions were nearly the same as solar terms were put near to those solar terms. The difference of mean motion in these cases is never greater than 20° in one period of analysis. When the difference is less than 7° a bracket has been put connecting the terms. In such cases even 50 years do not suffice thoroughly to disentangle the two terms, but there are only five brackets, one embracing three terms, to be considered. In the third line of the second column the value of $\frac{\theta}{\sin\theta}$ is given, 2θ being given in Table II. for the short-period terms. By this factor the coefficients in the next two columns must be multiplied before use.

The third and fourth columns give the "apparent" tabular minus observed coefficients of the sine and cosine of the argument. I say "apparent" because they have not been cleared of the effect of terms whose argument differs from that of the term considered either by an angle of long period or by the mean elongation D. In all cases a result is entered under (iii.) based on the observations 1847–1901. In about fifty cases, or one in three, results are given (i.) for periods 1 to 48, or 1750 to 1804 ; (ii.) for period 49 to 89, or 1804 to 1851 ; or (i.) and (ii.) a single result for 1750 to 1851. The result (iii.) is only comparable with (i.) or (ii.) after allowance for difference of tabular place.

The method of obtaining the quantities in the third or fourth

column has already been illustrated for the short-period terms with the mean anomaly as an example. For the terms of larger period see lxiv. June.

The fifth column gives the tabular coefficients (i.) Damoiseau's Tables modified by Airy and again by myself, see vol. lxiv. pp. 27–30, 571–573 ; (ii.) Hansen's Tables modified, see vol. lxiv. pp. 85, 414, 415.

The sixth column gives (i.) Hansen's theoretical coefficient, from the revised results in the "Darlegung" transformed by Professor Newcomb (*Astron. Papers Amer. Eph.*, vol. i.) ; (iii.) M. Radau's coefficient for planetary terms (Annales de l'Observatoire de Paris, *Mémoires*, xxi.), or in the case of four figure of Earth terms, Dr. G. W. Hill's coefficient (*Astron. Papers Amer. Eph.*, vol. iii.). These coefficients are marked R. H. respectively, to distinguish them from Hansen's solar terms.

The seventh column gives D_1 and D_2 from Professor Newcomb's "Transformation." They are Delaunay's coefficients before and after allowance for the higher terms not calculated.

The eighth column gives the observed coefficient. I only give them to 0″·1. From the third and fourth columns they can be deduced with a probable error of about 0″·05 ; but it did not seem worth while to apply probably erroneous corrections of less than 0″·1 to terms for which two independent theories in some cases agree to within 0″·01. Where the theoretical coefficients contain a secular term the observed coefficient in the eighth column is set down for the epoch 1875·0.

TABLE X.

Apparent Observed" Tabular and Theoretical Coefficients of 149 *Terms in the Moon's Longitude.*

fer-ice o.	Argument. $\frac{\theta}{\sin\theta}$	Apparent Tabular *minus* Observed Coefficients of sin	cos	Tabular Coefficients. (i.) Airy modified. (iii.) Hansen modified.	Theoretical Coefficients. Hansen. Radau (R). Hill (H).	D_1. D_2.	Concluded Observed Coefficients.
		(i.) periods 1–48 (ii.) „ 49–89 (iii.) „ 86–133					
		″	″	″	″	″	″
09	$2D-g+E-J$	...	...	0·00	...	...	...
	1·0	(iii.) −0·22	+0·04	0·00	+ 0·10R	...	+ 0·2
86	$g-g'+\omega-\omega'$	...	...	− 124·68	− 125·43	− 125·49	...
	D	...	...	...	...	− 125·98	...
	1·0	(iii.) 0·00	−0·14	− 124·90	...	...	− 124·9
91	$2g-2g'+2\omega-2\omega'$	...	...	+ 2370·10	+ 2369·75	+ 2369·74	...
	2D	...	...	...	...	+ 2369·74	...
	1·0	(iii.) −0·04	+0·05	+ 2370·14	...	...	+ 2370·2
95	$3g-3g'+3\omega-3\omega'$	...	...	+ 0·53	+ 0·41	+ 0·57	...
	3D	...	...	...	...	+ 0·54	...
	1·0	(iii.) −0·09	+0·01	+ 0·42	...	...	+ 0·5

Reference No.	Argument. $\frac{\theta}{\sin\theta'}$	Apparent Tabular *minus* Observed Coefficients of sin (i.) periods 1–48, (ii.) „ 49–89, (iii.) „ 86–133	cos	Tabular Coefficients. (i.) Airy modified. (iii.) Hansen modified.	Theoretical Coefficients. Hansen. Radau (R). Hill (H).	D_1. D_2.	Concluded Observed Coefficient.
98	$4g-4g'+4\omega-4\omega'$	...	...	+ 13″·94	+ 13″·90	+ 13″·89	...
	4D	...	...	...	...	+ 13·98	...
	1·0	(iii.) +0·09	−0·05	+ 13·90	...	...	+ 13·8
...	$5g-5g'+5\omega-5\omega'$	...	...	...	+ 0·01	+ 0·02	...
	5D	...	...	...	...	...	...
	1·0	(iii.) +0·08	−0·02	...	...	...	...
...	$5g-5g'+6\omega-6\omega'$	...	...	...	− 0·01	...	...
	$5D+\omega-\omega'$	...	...	...	...	...	...
	1·0	(iii.) 0·00	+0·03	...	...	...	...
96	$3g-3g'+4\omega-4\omega'$	...	...	− 0·90	− 0·64	− 0·83	...
	$3D+\omega-\omega'$	...	...	...	...	− 0·83	...
	1·0	(iii.) −0·01	−0·05	− 0·64	...	...	− 0·6
92	$2g-2g'+3\omega-3\omega'$	...	...	0·00	+ 0·28	+ 0·27	...
	$2D+\omega-\omega'$	...	...	...	...	...	...
	1·0	(iii.) +0·04	+0·09	+ 0·27	...	...	+ 0·2
87	$g-g'+2\omega-2\omega'$	...	...	− 28·80	− 28·56	− 29·50	...
	$D+\omega-\omega'$	...	...	...	...	− 28·32	...
	1·0	(iii.) +0·07	+0·04	− 28·60 + 0·07T	...	...	− 28·6
140	$\omega-\omega'$	(i.) −0·05	−0·15	+ 0·50	+ 1·33	+ 0·87	...
		(ii.) −0·35	−0·43	...	...	+ 0·87	...
	1·0	(iii.) +0·55	−0·36	+ 1·58	...	...	+ 1·0
85	$g-g'$	...	...	+ 148·10	+ 148·02	+ 147·46	...
	$D-\omega+\omega'$	...	...	...	...	+ 148·43	...
	1·0	(iii.) +0·03	−0·09	+ 148·03 − 0·28T.	...	...	+ 147·8
90	$2g-2g'+\omega-\omega'$	...	...	0·00	− 0·13	− 0·08	...
	$2D-\omega+\omega'$	...	...	...	...	...	...
	1·0	(iii.) +0·07	−0·01	− 0·13	...	...	− 0·2
94	$3g-3g'+2\omega-2\omega'$	...	...	+ 14·63	+ 14·60	+ 14·59	...
	$3D-\omega+\omega'$	...	...	...	...	+ 14·66	...
	1·0	(iii.) 0·00	+0·03	+ 14·60	...	...	+ 14·6

fer-ıce o.	Argument. $\frac{\theta}{\sin\theta}$	Apparent Tabular *minus* Observed Coefficients of sin (i.) periods 1–48 (ii.) „ 49–89 (iii.) „ 86–133	cos	Tabular Coefficients. (i.) Airy modified. (iii.) Hansen modified.	Theoretical Coefficients. Hansen. Radau (R). Hill (H).	D_1. D_2.	Concluded Observed Coefficients.
		"	"	"	"	"	"
00	$5g-5g'+4\omega-4\omega'$	...	...	0·00	+ 0·29	+ 0·20	...
	$5D-\omega+\omega'$	...	...	...	...	...	...
	1·0	(iii.) +0·08	−0·02	+ 0·29	...	...	+ 0·2
97	$3g-3g'+4\omega-2\omega'$	...	...	− 0·73	− 0·43	− 0·43	...
	$3D+\omega+\omega'$	...	...	...	...	− 0·43	...
	1·1	(iii.) +0·07	0·00	− 0·39	...	...	− 0·4
...	$3g-3g'+2\omega-4\omega'$	...	...	...	...	...	...
	$3D-\omega-\omega'$	...	...	...	...	...	...
	1·1	(iii.) +0·02	+0·02	...	...	...	...
...	$2g-2g'+4\omega-4\omega'$	...	...	...	− 0·03	− 0·01	...
	$2D+2\omega-2\omega'$	...	...	...	...	...	...
	1·1	(iii.) +0·02	−0·09	...	...	...	...
138	$2\omega-2\omega'$	(i.) −0·07	−0·29	0·00	− 0·23	− 0·16	...
		(ii.) +0·26	+0·05	...	...	...	...
	1·0	(iii.) −0·06	−0·01	− 0 27	...	...	− 0·2
89	$2g-2g'$	...	...	0·00	+ 0·19	+ 0·16	...
	$2D-2\omega+2\omega'$	...	...	...	...	...	...
	1·1	(iii.) 0·00	−0·01	+ 0·19	...	...	+ 0·2
99	$4g-4g'+6\omega-4\omega'$	...	...	0·00	− 0·17	− 0·14	...
	$4D+2\omega$	...	...	...	...	...	...
	1·3	(iii.) −0·04	+0·05	− 0·15	...	...	− 0·1
93	$2g-2g'+4\omega-2\omega'$	...	...	− 0·54	− 0·54	− 0·54	...
	$2D+2\omega$	...	...	...	...	− 0·53	...
	1·3	(iii.) +0·07	0·00	− 0·54	...	...	− 0·6
36	2ω	(i.) −0·39	+0·30	+ 1·10	+ 1·09	+ 1·39	...
		(ii.) −0·51	+0·05	...	...	+ 1·38	...
	1·1	(iii.) −0·15	+0·06	+ 1·25	...	...	+ 1·4
88	$2g-2g'-2\omega'$	...	...	− 0·50	− 0·43	− 0·45	...
	$2D-2\omega$	...	...	...	...	− 0·45	...
	1·3	(iii.) 0·00	−0·06	− 0·43	...	...	− 0·4

Reference No.	Argument. $\frac{\theta}{\sin\theta^\circ}$	Apparent Tabular *minus* Observed Coefficients of: sin (i.) periods (ii.) „ (iii.) „	cos 1–48 49–89 86–133	Tabular Coefficients. (i.) Airy modified. (iii.) Hansen modified.	Theoretical Coefficients. Hansen. Radau (R). Hill (H).	D_1. D_2.	Concluded Observed Coefficient.
		"	"	"	"	"	
...	$4g-4g'+2\omega-4\omega'$	...	...	...	− 0·01	− 0·01	...
	$4D-2\omega$	...	...	...	...	...	...
	1·3	(iii.) −0·02	+0·08	...	...	...	...
41	$4g-3g'+2\omega-2\omega'$	...	...	+ 1·16	+ 1·18	+ 1·11	...
	$4D+g'-2\omega+2\omega'$	...	...	...	...	+ 1·15	...
	1·1	(iii.) −0·03	+0·04	+ 1·18	...	...	+ 1·2
49	$2g-g'$	...	...	+ 9·60	+ 9·72	+ 9·59	...
	$2D+g'-2\omega+2\omega'$	...	...	...	...	+ 9·71	...
	1·2	(iii.) +0·02	−0·05	+ 9·72	...	...	+ 9·7
22	$g-\omega+\omega'$	(i.) +0·26	−0·04	0·00	− 0·38	− 0·26	...
	$D+g'-2\omega+2\omega'$	(ii.) +0·29	+0·01	...	...	...	...
	1·1	(iii.) −0·03	+0·02	− 0·38	...	...	− 0·3
134	$g'-2\omega+2\omega'$	(i.) +0·01	+0·09	− 2·40	− 2·52	− 1·87	...
		(ii.)		...	...	− 2·27	...
	1·0	(iii.) −0·04	+0·14	− 2·56	...	...	− 2·5
13	$2g-3g'+4\omega-4\omega'$	...	...	− 0·50	− 0·36	− 0·67	...
	$2D-g'+2\omega-2\omega'$	...	...	...	...	− 0·67	...
	1·0	(iii.) +0·02	−0·09	− 0·36	...	...	− 0·4
71	$5g-4g'+4\omega-4\omega'$	...	...	+ 1·93	+ 1·98	+ 1·86	...
	$5D+g'-\omega+\omega'$	...	...	...	...	+ 1·88	...
	1·2	(iii.) +0·07	+0·04	+ 1·98	...	...	+ 1·9
16	$3g-2g'+2\omega-2\omega'$	...	...	+ 192·10	+ 191·95	+ 192·00	...
	$3D+g'-\omega+\omega'$	...	...	...	...	+ 192·00	...
	1·1	(iii.) +0·03	+0·02	+ 191·94	...	...	+ 191·9
50	$2g-g'+\omega-\omega'$	...	...	− 8·20	− 8·48	− 8·45	...
	$2D+g'-\omega+\omega'$	...	...	...	...	− 8·54	...
	1·1	(iii.) −0·12	+0·05	− 8·54	...	...	− 8·5
23	g	(i.) −0·20	−0·20	+ 22639·50	+ 22640·15	+ 22640·15	...
	$D+g'-\omega+\omega'$	(ii.) −0·22	+1·14	...	...	+ 22640·15	...
	1·0	(iii.) +0·16	−0·16	+ 22639·62	...	...	+ 22639·5

Reference No.	Argument. $\frac{\theta}{\sin\theta}$	Apparent Tabular *minus* Observed Coefficients of	sin	cos	Tabular Coefficients. (i.) Airy modified. (iii.) Hansen modified.	Theoretical Coefficients. Hansen. Radau (R). Hill (H).	D_1. D_2.	Concluded Observed Coefficients.
			(i.) periods	1– 48				
			(ii.) "	49– 89				
			(iii.) "	86–133				
113	$g+2\varpi-3J+7^\circ$	(i.)	+0".13	−0".02	+ 0".32	...	...	...
		(ii.)	−0".17	−0".03	...	...	...	...
	1·0	(iii.)	−0".10	−0".13	+ 0".32	+ 0".32R	...	+ 0".4
132	$3V-4E-2^\circ$	(i.)			0".00	...	...	...
		(ii.)	+0".16	+0".27	...	...	...	...
	1·0	(iii.)	+0".10	+0".08	0".00	− 0".18R	...	− 0".1
131	$g'-\omega+\omega'$	(i.)			+ 18".60	+ 18".70	+ 18".35	...
		(ii.)	+0".17	−0".23	...	...	+ 18".76	...
	1·0	(iii.)	−0".08	+0".32	+ 18".60	...	...	+ 18".6
64	$g-2g'+2\omega-2\omega'$		...	...	+ 4586".34	+ 4586".56	+ 4586".24	...
	$D-g'+\omega-\omega'$		...	...	...	...	+ 4586".44	...
	1·0	(iii.)	+0".03	−0".31	+ 4586".44	...	...	+ 4586".4
12	$2g-3g'+3\omega-3\omega'$		...	...	− 3".00	− 3".23	− 2".98	...
	$2D-g'+\omega-\omega'$		...	...	...	...	− 3".12	..
	1·0	(iii.)	−0".06	+0".14	− 3".22	...	...	− 3".2
55	$3g-4g'+4\omega-4\omega'$		...	...	+ 38".46	+ 38".43	+ 38".31	...
	$3D-g'+\omega-\omega'$		...	...	...	...	+ 38".48	...
	1·0	(iii.)	−0".01	0".00	+ 38".43	...	...	+ 38".4
108	$2D-E+J$		...	...	0".00	...	...	...
	1·0	(iii.)	+0".23	−0".08	0".00	− 0".18R	...	− 0".2
83	$5g-6g'+6\omega-6\omega'$		...	...	0".00	+ 0".40	+ 0".26	...
	$5D-g'+\omega-\omega'$		...	...	...	...	...	...
	1·5	(iii.)	−0".06	−0".02	+ 0".40	...	...	+ 0".4
42	$4g-3g'+4\omega-4\omega'$		...	...	− 0".20	− 0".29	− 0".29	...
	$4D+g'$		...	...	...	...	− 0".30	...
	1·0	(iii.)	+0".04	+0".13	− 0".29	...	...	− 0".3
17	$3g-2g'+3\omega-3\omega'$		...	...	0".00	+ 0".15	+ 0".14	...
	$3D+g'$		...	...	...	...	...	...
	1·0	(iii.)	−0".20	−0".20	+ 0".15	...	...	+ 0".3

Reference No.	Argument. $\frac{\theta}{\sin\theta}$	Apparent Tabular *minus* Observed Coefficients of sin (i.) periods (ii.) „ (iii.) „	cos 1–48 49–89 86–133	Tabular Coefficients. (i.) Airy modified. (iii.) Hansen modified.	Theoretical Coefficients. Hansen. Radau (R). Hill (H).	D_1. D_2.	Concluded Observed Coefficient
		"	"	"	"	"	
51	$2g-g'+2\omega-2\omega'$	...	...	− 24·48	− 24·45	− 24·60	...
	$2D+g'$	...	...	....	...	− 24·50	...
	1·0	(iii.) −0·17	+0·02	− 24·45	...	...	− 2
				+ 0·06T			
24	$g+\omega-\omega'$	(i.) +0·27	−0·04	+ 17·90	+ 18·09	+ 18·08	...
	$D+g'$	(ii.) +0·89	+0·02	...	...	+ 18·08	...
	1·0	(iii.) +0·70	+0·29	+ 18·17	...	...	+ 1
107	$2D-g+3V-3E$	(i.) +0·58	+0·02	0·00	...	...	...
		(ii.) +0·90	−0·30	...	...	...	...
	1·0	(iii.) −0·39	−0·27	− 0·68	− 0·68R	...	− C
128	g'	(i.)		− 669·63	− 669·85	− 669·57	...
		(ii.) −0·17	−0·10	+ 1·63T	...	− 669·76	...
	1·0	(iii.) −0·38	−0·28	− 669·46	...	...	− 668
				+ 1·64T			
106	$g-3V+3E$	...	...	0·00	...	...	...
	1·0	(iii.) +0·08	+0·01	0·00	− 0·14R	...	− C
63	$g-2g'+\omega-\omega'$	...	...	− 0·40	− 0·60	− 0·55	...
	$D-g'$	...	...	...	...	− 0·56	...
	1·0	(iii.) +0·04	+0·05	− 0·59	...	...	− C
11	$2g-3g'+2\omega-2\omega'$	...	...	+ 165·52	+ 165·52	+ 165·55	...
	$2D-g'$	...	...	...	...	+ 165·55	...
	1·0	(iii.) +0·07	+0·02	+ 165·56	...	...	+ 165
				− 0·39T			
70	$4g-5g'+4\omega-4\omega'$	...	...	+ 1·78	+ 1·89	+ 1·67	...
	$4D-g'$	...	...	...	...	+ 1·71	...
	1·1	(iii.) −0·01	−0·07	+ 1·89	...	...	+ 1
43	$4g-3g'+4\omega-2\omega'$	...	...	0·00	− 0·38	− 0·37	...
	$4D+g'+2\omega'$	...	...	...	...	...	...
	1·1	(iii.) −0·02	+0·02	− 0·38	...	...	− C
127	$g'+2\omega'$	(i.) +0·21	−0·06	+ 1·50	+ 1·55	+ 1·43	...
		(ii.)		...	...	+ 1·43	...
	1·0	(iii.) +0·11	−0·02	+ 1·50	...	...	+ 1

fer-ice fo.	Argument. $\frac{\theta}{\sin\theta}$	Apparent Tabular *minus* Observed Coefficients of sin (i.) periods (ii.) „ (iii.) „	cos 1–48 49–89 86–133	Tabular Coefficients. (i.) Airy modified. (iii.) Hansen modified.	Theoretical Coefficients. Hansen. Radau (R). Hill (H).	D_1. D_2.	Concluded Observed Coefficients.
25	$g+2\omega-2\omega'$	(i.) 0″·00	−0″·15	− 2″·45	− 2″·54	− 2″·22	... ″
	$D+g'+\omega-\omega'$	(ii.) +0·06	+0·12	...	...	− 2·35	...
	1·1	(iii.) −0·06	−0·01	− 2·49	...	...	− 2·4
62	$g-2g'$	...	...	+ 2·59	+ 2·59	+ 2·49	...
	$D-g'-\omega+\omega'$	...	...	...	...	+ 2·59	...
	1·0	(iii.) +0·05	−0·04	+ 2·57	...	...	+ 2·5
54	$3g-4g'+2\omega-2\omega'$	...	...	+ 0·74	+ 0·76	+ 0·68	...
	$3D-g'-\omega+\omega'$	...	...	...	...	+ 0·72	...
	1·1	(iii.) +0·03	−0·04	+ 0·76	...	...	+ 0·7
72	$5g-4g'+6\omega-4\omega'$	...	...	0·00	− 0·20	− 0·16	...
	$5D+g'+\omega+\omega'$	...	...	...	...	...	...
	1·0	(iii.) +0·03	−0·02	− 0·20	...	...	− 0·2
18	$3g-2g'+4\omega-2\omega'$	...	...	− 9·40	− 9·37	− 9·34	...
	$3D+g'+\omega+\omega'$	...	...	...	...	− 9·39	...
	1·1	(iii.) +0·03	−0·05	− 9·37	...	...	− 9·4
26	$g+2\omega$	(i.) +0·08	−0·08	− 39·56	− 39·58	− 39·54	...
	$D+g'+\omega+\omega'$	(ii.) +0·01	−0·32	...	...	− 39·54	...
	1·2	(iii.) +0·14	−0·05	− 39·28	...	...	− 39·5
61	$g-2g'-2\omega'$	...	...	− 6·20	− 6·36	− 6·37	...
	$D-g'-\omega-\omega'$	...	...	...	...	− 6·37	...
	1·1	(iii.) +0·07	+0·05	− 6·44	...	...	− 6·5
73	$3g-g'$	...	...	+ 0·40	+ 0·67	+ 0·63	...
	$3D+2g'-3\omega+3\omega'$	..	...	...	...	+ 0·66	...
	1·6	(iii.) −0·03	+0·02	+ 0·67	...	...	+ 0·7
7	$g+g'-2\omega+2\omega'$	...	...	0·00	− 0·18	− 0·07	...
	$D+2g'-3\omega+3\omega'$	...	...	...	...	...	...
	1·2	(iii.) −0·02	+0·01	− 0·12	...	...	− 0·1
21	$6g-4g'+4\omega-4\omega'$	...	...	0·00	+ 0·22	+ 0·18	...
	$6D+2g'-2\omega+2\omega'$	...	...	...	...	+ 0·20	...
	1·2	(iii.) −0·05	+0·04	+ 0·17	...	...	+ 0·2

Reference No.	Argument. $\frac{\theta}{\sin\theta}$	Apparent Tabular *minus* Observed Coefficients of sin	cos	Tabular Coefficients. (i.) Airy modified. (iii.) Hansen modified.	Theoretical Coefficients. Hansen. Radau (R). Hill (H).	D_1. D_2.	Concluded Observed Coefficient
		(i.) periods	1– 48				
		(ii.) „	49– 89				
		(iii.) „	86–133				
		″	″	″	″	″	″
52	$4g-2g'+2\omega-2\omega'$	...	...	+ 14·10	+ 14·38	+ 14·40	...
	$4D+2g'-2\omega+2\omega'$	...	...	...	...	+ 14·40	...
	1·4	(iii.) +0·01	+0·01	+ 14·37	...	...	+ 1
74	$3g-g'+\omega-\omega'$	...	...	− 0·40	− 0·59	− 0·57	...
	$3D+2g'-2\omega+2\omega'$	...	...	...	...	− 0·60	...
	1·3	(iii.) −0·07	+0·06	− 0·59	...	...	− 0
27	$2g$	(i.) +0·15	−0·04	+ 769·04	+ 769·06	+ 769·12	...
	$2D+2g'-2\omega+2\omega'$	(ii.) +0·24	+0·08	...	...	+ 769·06	...
	1·0	(iii.) −0·01	−0·08	+ 769·06	...	...	+ 76
8	$g+g'-\omega+\omega'$	...	...	+ 1·58	+ 1·78	+ 1·50	...
	$D+2g'-2\omega+2\omega'$	...	...	...	...	+ 1·59	...
	1·1	(iii.) +0·18	−0·04	+ 1·80	...	...	+
125	$2g'-2\omega+2\omega'$	(i.)		− 211·75	− 211·71	− 211·46	...
		(ii.) −0·11	−0·07	...	...	− 211·84	...
	1·1	(iii.) −0·12	+0·01	− 211·70	...	...	− 21
59	$g-3g'+3\omega-3\omega'$	...	...	− 1·21	− 1·22	− 1·17	...
	$D-2g'+2\omega-2\omega'$	...	...	...	...	− 1·23	...
	1·1	(iii.) −0·02	−0·02	− 1·25	...	...	−
110	$2D-g-E+J$	...	...	...	...	...	...
	1·1	(iii.) +0·15	+0·01	...	− 0·22R	...	− 0
67	$2g-4g'+4\omega-4\omega'$	...	...	+ 30·75	+ 30·78	+ 30·52	...
	$2D-2g'+2\omega-2\omega'$	...	...	...	...	+ 30·72	...
	1·1	(iii.) −0·01	0·00	+ 30·78	...	...	+ 30
20	$4g-6g'+6\omega-6\omega'$	...	...	+ 0·54	+ 0·57	+ 0·40	...
	$4D-2g'+2\omega-2\omega'$	...	...	...	...	+ 0·51	...
	1·0	(iii.) 0·00	+0·02	+ 0·57	...	...	+ 0
75	$3g-g'+2\omega-2\omega'$	...	...	− 2·90	− 2·93	− 2·96	...
	$3D+2g'-\omega+\omega'$	...	...	...	...	− 2·96	...
	1·1	(iii.) −0·06	−0·16	− 2·93	...	...	− 2

er-ce).	Argument. $\frac{\theta}{\sin\theta}$	Apparent Tabular *minus* Observed Coefficients of sin (i.) periods 1–48, (ii.) „ 49–89, (iii.) „ 86–133	cos	Tabular Coefficients. (i.) Airy modified. (iii.) Hansen modified.	Theoretical Coefficients. Hansen. Radau (R). Hill (H).	D_1. D_2.	Concluded Observed Coefficients.
28	$2g+\omega-\omega'$	(i.) −0″·32	+0″·10	+ 1″·00	+ 1″·27	+ 1″·22	... ″
	$2D+2g'-\omega+\omega'$	(ii.) −0·27	+0·24	...	...	+ 1·21	...
	1·0	(iii.) −0·04	+0·21	+ 1·28	...	...	+ 1·
9	$g+g'$	...	...	− 109·90	− 109·92	− 109·79	...
	$D+2g'-\omega+\omega'$	...	...	...	...	− 109·85	...
	1·0	(iii.) +0·10	−0·04	− 109·95 + 0·18T	...	...	− 109·9
23	$2g'-\omega+\omega'$	(i.) −0·05	−0·08	0·00	+ 0·17	+ 0·14	...
		(ii.)		...	...	+ 0·14	...
	1·1	(iii.) +0·09	−0·08	+ 0·15	...	...	+ 0·1
58	$g-3g'+2\omega-2\omega'$	...	...	+ 206·40	+ 206·46	+ 206·54	...
	$D-2g'+\omega-\omega'$	...	...	...	...	+ 206·34	...
	1·0	(iii.) +0·05	+0·10	+ 206·49 − 0·49T	...	...	+ 206·1
56	$2g-4g'+3\omega-3\omega'$	...	...	0·00	− 0·23	− 0·18	...
	$2D-2g'+\omega-\omega'$	...	...	...	...	...	...
	1·0	(iii.) +0·08	−0·06	− 0·23	...	...	− 0·3
3	$3g-5g'+4\omega-4\omega'$	...	...	+ 4·38	+ 4·41	+ 4·28	...
	$3D-2g'+\omega-\omega'$	...	...	...	...	+ 4·34	...
	1·0	(iii.) −0·02	+0·01	+ 4·40	...	...	+ 4·4
76	$3g-g'+2\omega$	...	...	0·00	− 0·30	− 0·28	...
	$3D+2g'-\omega+3\omega'$	...	...	...	...	...	...
	1·0	(iii.) +0·10	+0·15	− 0·30	...	...	− 0·4
29	$2g+2\omega-2\omega'$	(i.) −0·16	+0·10	0·00	− 0·19	− 0·15	...
	$2D+2g'$	(ii.) +0·32	−0·34	...	...	− 0·15	...
	1·0	(iii.) +0·20	−0·32	− 0·19	...	...	− 0·2
22	$2g'$	(i.) −0·13	−0·08	− 7·50	− 7·51	− 7·49	...
		(ii.)		...	...	7·46	...
	1·1	(iii.) +0·05	−0·04	− 7·49	...	...	− 7·5
65	$2g-4g'+2\omega-2\omega'$	...	...	+ 7·80	+ 8·13	+ 8·06	...
	$2D-2g'$	...	...	...	...	+ 8·06	...
	1·0	(iii.) +0·01	−0·02	+ 8·12	...	...	+ 8·1

Reference No.	Argument. $\frac{\theta}{\sin\theta}$	Apparent Tabular *minus* Observed Coefficients of sin (i.) periods (ii.) „ (iii.) „	cos 1– 48 49– 89 86–133		Tabular Coefficients. (i.) Airy modified. (iii.) Hansen modified.		Theoretical Coefficients. Hansen. Radau (R). Hill (H).		D_1. D_2.		Concluded Observed Coefficient
		"	"		"		"		"		
19	$4g-6g'+4\omega-4\omega'$	...	...		0·00	+	0·15	+	0·10		...
	$4D-2g'$	...	...		...		...		...		...
	1·1	(iii.) −0·06	+0·03	+	0·15		...		...	+	0
53	$4g-2g'+4\omega-2\omega'$	...	...	−	5·74	−	5·74	−	5·73		...
	$4D+2g'+2\omega'$	...	...		...		...	−	5·73		...
	1·0	(iii.) +0·08	+0·02	−	5·75		...		...	−	5
77	$3g-g'+3\omega-\omega'$	...	...		0·00	+	0·25	+	0·24		...
	$3D+2g'+2\omega'$	...	...		...		...		...		...
	1·0	(iii.) −0·07	+0·06	+	0·24		...		...	+	0
30	$2g+2\omega$	(i.) +0·10	0·00	−	411·62	−	411·60	−	411·63		...
	$2D+2g'+2\omega'$	(ii.) −0·06	−0·04		...		...	−	411·63		...
	1·1	(iii.) +0·01	+0·11	−	411·60		...		...	−	411
10	$g+g'+\omega+\omega'$	...	...	+	0·70	+	0·55	+	0·59		...
	$D+2g'+2\omega'$	...	...		...		...	+	0·59		...
	1·1	(iii.) −0·24	+0·02	+	0·57		...		...	+	0
121	$2g'+2\omega'$	(i.)		−	55·20	−	55·25	−	55·20		...
		−0·01	−0·03								
		(ii.)			...		...	−	55·17		...
	1·1	(iii.) +0·25	−0·02	−	54·98		...		...	−	55
57	$g-3g'+\omega-3\omega'$	...	...	−	0·20	−	0·32	−	0·26		..
	$D-2g'-2\omega'$	...	...		...		...	−	0·25		...
	1·0	(iii.) +0·03	+0·08	−	0·30		...		...	−	0
105	$2D-g-2V+2E$	...	...		...		...		...		...
	1·0	(iii.) +0·12	−0·02		...	−	0·17R		...	−	0
56	$g-3g'-2\omega'$	...	...	−	0·68	−	0·40	−	0·37		...
	$D-2g'-\omega-\omega'$	...	...		...		...	−	0·37		...
	1·1	(iii.) +0·03	+0·04	−	0·34		...		...	−	0
44	$5g-2g'+2\omega-2\omega'$	...	...	+	0·80	+	1·06	+	1·06		...
	$5D+3g'-3\omega+3\omega'$	...	...		...		...	+	1·06		...
	1·2	(iii.) +0·03	+0·04	+	1·06		...		...	+	1

Reference No.	Argument. $\frac{\theta}{\sin\theta}$	Apparent Tabular *minus* Observed Coefficients of	sin	cos	Tabular Coefficients. (i.) Airy modified. (iii.) Hansen modified.	Theoretical Coefficients. Hansen. Radau (R). Hill (H).	D_1. D_2.	Concluded Observed Coefficients.
			(i.) periods	1– 48				
			(ii.) "	49– 89				
			(iii.) "	86–133				
31	$3g$	(i.)	+0".05	−0".03	+ 36".12	+ 36".13	+ 36".16	... "
	$3D+3g'-3\omega+3\omega'$	(ii.)	+0·09	+0·06	...	...	+ 36·12	...
	1·1	(iii.)	−0·09	−0·08	+ 36·13	...	...	+ 36·2
38	$2g+g'-\omega+\omega'$		...	...	0·00	+ 0·12	+ 0·09	...
	$2D+3g'-3\omega+3\omega'$		...	...	...	...	...	...
	1·2	(iii.)	−0·01	+0·10	+ 0·12	...	...	+ 0·1
80	$g+2g'-2\omega+2\omega'$		...	...	− 13·24	− 13·19	− 13·15	...
	$D+3g'-3\omega+3\omega'$		...	...	...	...	− 13·32	...
	1·2	(iii.)	+0·04	−0·07	− 13·20	...	...	− 13·2
6	$g-4g'+4\omega-4\omega'$		...	...	+ 1·10	+ 1·18	+ 0·96	...
	$D-3g'+3\omega-3\omega'$		...	...	...	...	+ 1·08	...
	1·1	(iii.)	−0·07	+0·02	+ 1·18	...	...	+ 1·2
69	$3g-6g'+6\omega-6\omega'$		...	...	0·00	+ 0·29	+ 0·20	...
	$3D-3g'+3\omega-3\omega'$		...	...	...	...	...	...
	1·2	(iii.)	−0·01	+0·04	+ 0·29	...	...	+ 0·3
78	$4g-g'+2\omega-2\omega'$		...	...	0·00	− 0·29	− 0·27	...
	$4D+3g'-2\omega+2\omega'$		...	...	...	...	...	...
	1·0	(iii.)	+0·03	+0·04	− 0·29	...	...	− 0·3
39	$2g+g'$		...	...	− 7·68	− 7·67	− 7·62	...
	$2D+3g'-2\omega+2\omega'$		...	...	...	...	− 7·69	...
	1·1	(iii.)	−0·02	+0·01	− 7·67	...	...	- 7·7
120	$3g'-2\omega+2\omega'$	(i.)	−0·03	−0·07	− 8·66	− 8·66	− 8·66	...
		(ii.)			...	...	− 8·66	...
	1·2	(iii.)	0·00	−0·01	− 8·69	...	...	− 8·7
2	$2g-5g'+4\omega-4\omega'$		...	...	+ 2·75	+ 2·75	+ 2·69	...
	$2D-3g'+2\omega-2\omega'$		...	...	...	...	+ 2·75	...
	1·0	(iii.)	−0·04	+0·07	+ 2·75	...	...	+ 2·8
81	$g+2g'$		...	...	-- 1·20	− 1·18	− 1·16	...
	$D+3g'-\omega+\omega'$		...	...	...	...	− 1·16	...
	1·0	(iii.)	−0·11	+0·01	− 1·18	...	...	− 1·1

Reference No.	Argument. $\frac{\theta}{\sin\theta}$.	Apparent Tabular *minus* Observed Coefficients of		Tabular Coefficients. (i.) Airy modified. (iii.) Hansen modified.	Theoretical Coefficients. Hansen. Radau (R). Hill (H).	D_1. D_2.	Concluded Observed Coefficients
		sin	cos				
		(i.) periods	1– 48				
		(ii.) ,,	49– 89				
		(iii.) ,,	86–133				
5	$g-4g'+2\omega-2\omega'$	...	...	+ 7″·50	+ 7″·44	+ 7″·50	...
	$D-3g'+\omega-\omega'$	...	...	...	...	+ 7·50	...
	1·0	(iii.) +0·01	+0·05	+ 7·42	...	...	+ ·4
68	$3g-6g'+4\omega-4\omega'$	...	...	0·00	+ 0·31	+ 0·22	...
	$3D-3g'+\omega-\omega'$	...	...	...	...	...	...
	1·0	(iii.) 0·00	+0·03	+ 0·31	...	...	+ 0·3
45	$5g-2g'+4\omega-2\omega'$	...	...	− 1·00	− 0·99	− 0·98	...
	$5D+3g'-\omega+3\omega'$	...	...	...	...	− 1·00	...
	1·0	(iii.) −0·03	−0·03	− 0·99	...	...	− 1·0
32	$3g+2\omega$	(i.) +0·25	0·00	− 45·20	− 45·09	− 45·12	...
	$3D+3g'-\omega+3\omega'$	(ii.) −0·05	+0·31	...	...	− 45·12	...
	1·0	(iii.) −0·10	−0·02	− 45·08	...	...	− 45·2
82	$g+2g'+2\omega'$	...	...	0·00	− 0·18	− 0·18	...
	$D+3g'-\omega+3\omega'$	...	...	...	...	− 0·14	...
	1·0	(iii.) +0·05	+0·07	− 0·14	...	...	− 0·2
4	$g-4g'+2\omega-4\omega'$	...	...	0·00	+ 0·22	+ 0·34	...
	$D-3g'+\omega-3\omega'$	...	...	...	...	+ 0·34	...
	1·1	(iii.) −0·13	−0·03	+ 0·20	...	...	+ 0·3
119	$3g'$	(i.)		0·00	− 0·08	− 0·14	...
		(ii.) +0·07	+0·06	...	...	...	...
	1·2	(iii.) −0·07	−0·08	...	...	...	0·0
1	$2g-5g'+2\omega-2\omega'$	...	...	0·00	+ 0·34	+ 0·25	...
	$2D-3g'$	...	...	...	...	..	...
	1·2	(iii.) +0·04	−0·03	+ 0·34	...	...	+ 0·3
40	$2g+g'+2\omega$	...	...	+ 1·02	+ 0·42	+ 0·42	...
	$2D+3g'+2\omega'$	...	...	...	...	+ 0·42	...
	1·1	(iii.) 0·00	0·00	+ 0·42	...	...	+ 0·4
118	$3g'+2\omega'$	(i.)		− 2·30	− 2·15	− 2·17	...
		(ii.) −0·12	−0·07	...	...	− 2·17	...
	1·2	(iii.) +0·01	−0·02	− 2·15	...	...	− 2·2

Reference No.	Argument. $\frac{\theta}{\sin\theta}$	Apparent Tabular *minus* Observed Coefficients of sin (i.) periods 1–48, (ii.) " 49–89, (iii.) " 86–133	cos	Tabular Coefficients. (i.) Airy modified. (iii.) Hansen modified.	Theoretical Coefficients. Hansen. Radau (R). Hill (H).	D_1.	D_2.	Concluded Observed Coefficients.
33	$4g$	(i.) −0".14	+0".03	+ 2".00	+ 1".94	+	1".96	... "
	$4D+4g'-4\omega+4\omega'$	(ii.) +0·09	−0·07	...	...	+	1·94	...
	1·1	(iii.) +0·06	0·00	+ 1·93	...		...	+ 1·9
14	$2g+2g'-2\omega+2\omega'$	...	...	− 0·90	− 0·95	−	0·91	...
	$2D+4g'-4\omega+4\omega'$	...	...	...	...	−	1·00	...
	1·3	(iii.) −0·01	+0·05	− 0·95	...		...	− 0·9
46	$3g+g'$	...	...	− 0·40	− 0·55	−	0·52	...
	$3D+4g'-3\omega+3\omega'$	...	...	...	...	−	0·56	...
	1·2	(iii.) +0·06	+0·01	− 0·55	...		...	− 0·6
84	$g+3g'-2\omega+2\omega'$	...	...	− 0·30	− 0·48	−	0·49	...
	$D+4g'-3\omega+3\omega'$	...	...	...	...	−	0·49	...
	1·0	(iii.) −0·01	+0·03	− 0·45	...		...	− 0·4
117	$4g'-2\omega+2\omega'$	(i.)		0·00	− 0·28	−	0·28	...
		(ii.) +0·15	−0·01	...	...		...	...
	1·4	(iii.) +0·21	−0·02	− 0·13	...		...	− 0·3
60	$2g-6g'+4\omega-4\omega'$	...	...	0·00	+ 0·16	+	0·11	...
	$2D-4g'+2\omega-2\omega'$	...	...	...	...		...	...
	1·1	(iii.) +0·06	+0·07	+ 0·16	...		...	+ 0·
79	$6g-2g'+4\omega-2\omega'$	...	...	0·00	− 0·12	−	0·12	...
	$6D-4g'-2\omega+4\omega'$	...	...	...	...		...	...
	1·3	(iii.) −0·01	0·00	− 0·12	...		...	− 0·1
34	$4g+2\omega$	(i.) −0·14	−0·10	− 4·10	− 4·00	−	4·01	...
	$4D+4g'-2\omega+4\omega'$	(ii.) −0·02	+0·06	...	...	−	4·01	...
	1·0	(iii.) −0·01	+0·07	− 4·00	...		...	− 4·0
15	$2g+2g'+2\omega'$	...	...	+ 0·55	+ 0·56	+	0·54	...
	$2D+4g'-2\omega+4\omega'$	...	...	...	...	+	0·54	...
	1·0	(iii.) +0·03	−0·07	+ 0·56	...		...	+ 0·5
48	$g-5g'+2\omega-2\omega'$	...	...	0·00	+ 0·26	+	0·19	...
	$D-4g'+\omega-\omega'$	...	...	...	...		...	...
	1·0	(iii.) −0·04	0·00	+ 0·26	...		...	+ 0·3

Reference No.	Argument. $\frac{\theta}{\sin\theta}$	Apparent Tabular *minus* Observed Coefficients of sin (i.) periods 1– 48 (ii.) „ 49– 89 (iii.) „ 86–133	cos	Tabular Coefficients. (i.) Airy modified. (iii.) Hansen modified.	Theoretical Coefficients. Hansen. Radau (R). Hill (H).	D_1. D_2.	Concluded Observed Coefficients.
		"	"	"	"	"	"
47	$3g+g'+2\omega$	...	...	0·00 +	0·27 +	0·26	...
	$3D+4g'-\omega+3\omega'$	...	...	...	...	...	...
	1·0	(iii.) +0·05	+0·14 +	0·27	...	...	+ 0·2
35	$4g+4\omega$	(i.) +0·06	−0·07 +	0·40 +	0·42 +	0·42	...
	$4D+4g'+4\omega'$	(ii.) −0·06	−0·07	...	... +	0·42	...
	1·5	(iii.) −0·03	0·00 +	0·42	...	...	+ 0·4
36	$5g$	(i.) −0·01	+0·04	+0·10 +	0·11 +	0·12	...
	$5D+5g'-5\omega+5\omega'$	(ii.) +0·09	+0·12	...	... +	0·11	...
	1·2	(iii.) +0·03	−0·10 +	0·11	...	...	+ 0·1
37	$5g+2\omega$	(i.) −0·07	+0·04 −	0·30 −	0·33 −	0·33	...
	$5D+5g'-3\omega+5\omega'$	(ii.) −0·08	0·00	...	... −	0·33	...
	1·0	(iii.) +0·02	+0·05 −	0·33	...	...	− 0·3
101	$2D-V+E$	...	...	...	...	...	...
	1·2	(iii.) −0·15	0·00	... +	0·10R	...	+ 0·1
102	$2D-g-V+E$	...	...	...	...	...	...
	1·7	(iii.) −0·05	−0·02	... +	0·14R	...	+ 0·1
103	$2D-2V+2E$	...	...	...	...	...	...
	1·3	(iii.) +0·13	+0·02	... −	0·15R	...	− 0·1
104	$2D-g+2V-2E$	...	...	...	...	...	...
	1·3	(iii.) +0·03	−0·02	... +	0·19R	...	0·0
111	$g-2E+2J$	...	...	...	...	...	...
	1·0	(iii.) +0·26	−0·02	... −	0·16R	...	− 0·3
115	$g+\Omega$	(i.) −0·39	0·00	...	...	...	...
		(ii.) −0·41	−0·03	...	...	...	...
	1·0	(iii.) −0·11	+0·12 +	0·52 +	0·52H	...	+ 0·5
116	$2g+2\omega+\Omega$	(i.) −0·07	−0·06 +	0·30	...	...	...
	$2g+2\varpi-\Omega$	(ii.) +0·01	−0·08	...	...	...	...
	1·1	(iii.) +0·02	+0·08 +	0·39 +	0·39R	...	+ 0·4

Reference No.	Argument. $\frac{\theta}{\sin\theta^{\circ}}$	Apparent Tabular *minus* Observed Coefficients of sin / cos		Tabular Coefficients. (i.) Airy modified. (iii.) Hansen modified.	Theoretical Coefficients. Hansen. Radau (R). Hill (H).	D_1. D_2.	Concluded Observed Coefficients.
		(i.) periods	1– 48				
		(ii.) ,,	49– 89				
		(iii.) ,,	86–133				
112	2D − *g* + 2E − 2J	(i.) + 0″·13	+ 0″·46	− 0″·89	...	...	...
		(ii.) + 0·55	+ 0·06	...	...	...	...
	1·0	(iii.) + 0·24	+ 0·03	− 0·89	− 0·88R	...	− 1·1
114	*g* − ☊	(i.) + 0·51	+ 0·09	...	...	...	...
		(ii.) + 0·57	+ 0·15	...	...	...	...
	1·0	(iii.) + 0·14	+ 0·19	− 0·52	− 0·52H	...	− 0·5
129	2E − 2M	(i.) (ii.) − 0·30	+ 0·01	...	...	...	...
	1·0	(iii.) − 0·06	0·00	+ 0·24	+ 0·23R	...	+ 0·3
130	E − J	(i.) (ii.) + 0·02	− 0·07	+ 0·70	...	...	...
	1·0	(iii.) + 0·01	− 0·13	+ 0·74	+ 0·65R	...	+ 0·7
124	2E − 2J	(i.) (ii.) + 0·09	− 0·01	− 0·20	...	...	...
	1·1	(iii.) − 0·05	− 0·05	− 0·24	− 0·20R	...	− 0·2
133	E − 2J + 298°	(i.) (ii.) − 0·24	− 0·06	...	...	...	...
	1·0	(iii.) − 0·16	+ 0·04	...	+ 0·17R	...	+ 0·2
134	V − E	(i.) (ii.) − 0·05	− 0·11	− 0·80	...	...	...
	1·0	(iii.) − 0·34	− 0·02	− 1·10	− 0·86R	...	− 0·8
126	2V − 2E	(i.) (ii.) + 0·11	+ 0·02	+ 0·40	...	...	...
	1·0	(iii.) + 0·14	− 0·02	+ 0·43	+ 0·28R	...	+ 0·3
137	2V − 3E − 5°	(i.) + 0·05	+ 0·14	− 0·30 cos	...	...	...
		(ii.) − 0·13	+ 0·10	...	...	...	...
	1·1	(iii.) + 0·22	+ 0·24	...	− 0·35R cos	...	− 0·3 cos
141	J + 9°	(i.) + 0·44	+ 0·50	...	...	...	...
		(ii.) − 0·15	+ 0·19	...	...	...	...
	1·0	(iii.) − 0·06	+ 0·14	...	− 0·14R cos	...	− 0·1 cos

Reference No.	Argument. $\frac{\theta}{\sin\theta}$	Apparent Tabular *minus* Observed Coefficients of sin / cos	Tabular Coefficients (i.) Airy modified. (iii.) Hansen modified.	Theoretical Coefficients. Hansen. Radau (R). Hill (H).	D_1. D_2.	Concluded Observed Coefficients.
		(i.) periods 1–48; (ii.) „ 49–89; (iii.) „ 86–133				
		" "	"	"	"	"
139	$4M-2E+98°$	(i.) +0·12 +0·33	...	...	...	...
		(ii.) +0·13 −0·18	...	...	...	...
	1·0	(iii.) −0·04 −0·02	− 0·05 sin	− 0·09R sin	...	− 0·1 sin
			+ 0·09 cos	+ 0·06R cos	...	+ 0·1 cos
142	$2M-E+49°$	(i.) +0·67 +0·50	...	...	...	...
		(ii.) +0·52 −0·50	...	...	...	...
	1·0	(iii.) −0·41 −0·12	− 0·32	− 0·42R	...	− 0·4
143	$2\varpi-2J$	(i.) −0·67 +0·87	...	...	...	...
		(ii.) −0·39 +0·62	...	...	...	...
	1·0	(iii.) −0·59 +0·73	...	+ 0·21R	...	+ 0·2
144	$-\Omega$	(i.) +0·35 +0·97	− 6·60	...	...	...
		(ii.) −0·21 +0·66	...	...	...	...
	1·0	(iii.) −1·19 −0·18	− 7·34 sin	− 7·67H	...	− 6·6 sin
			− 0·71 cos			− 0·7 cos

I add a few remarks on Table X.

If columns 80^s3, 80^c3, 53^s2, 53^c2, sin D, cos D, sin 2D, cos 2D, μ, and of course the column indicating the number of observations in Table VIII., be suppressed, we are left with 74×48 quantities, which have a mean value, without regard to sign, of $\pm 0''{\cdot}22$. From this a probable error of $\pm 0''{\cdot}22 \times 0{\cdot}85 = \pm 0''{\cdot}19$ may be inferred, and consequently a probable error of $\pm 0''{\cdot}19 \div \sqrt{48} = \pm 0''{\cdot}027$ for the quantities in third and fourth columns of Table X., line (iii.). We may expect, therefore, and in fact we find, that apparent cosine terms of over $0''{\cdot}1$, and apparent sine terms differing by $0''{\cdot}1$ from theory are exceedingly rare. Quantities of $0''{\cdot}2$ in the case of short-period terms may be regarded as almost certainly real, whether an explanation of them is forthcoming or not.

Now taking the entries of Table X. in the order of that table :

91. The observed variation is larger than the theoretical. As Professor Newcomb has explained, this may be attributed to the conditions of observation (" Corrections to Hansen's Tables," 1876, p. 8).

96. A term with characteristic ee', where Hansen's coefficient is clearly preferable to Delaunay's.

87. Same remark as 96.

140. A term of nine years' period. Less accuracy may be expected in the observed coefficient. Delaunay's value is preferable to Hansen's.

85. Same remark as 96.

138. The first fifty years of the Airy period in many cases seem less accurate than the second fifty years. See term 141.

136. A similar remark to 140. Period, three years.

134.
13. } Similar remark as 96 ; characteristic e^2e'.

23. The principal elliptic inequality. This must be considered in connection with the terms 50 and 131. If we ignore these "allied" terms, the principal elliptic inequality would seem to be $22639''\cdot71$ from Airy and $22639''\cdot46$ from Hansen, a difference of $0''\cdot25$. But in term 50 Airy's tabular place exceeds Hansen's by $+0''\cdot34 \sin(g+\mathrm{D})$, which for a "$g$" analysis produces the apparent effect of $-0''\cdot17 \sin g$. This reduces the discordance to $0''\cdot08$. The full statement for the value of the principal elliptic inequality is this : Let $-8''\cdot54+\alpha$ and $+18''\cdot60+\beta$ be the true coefficients of $\sin(g+\mathrm{D})$ and $\sin(g-\mathrm{D})$ respectively, corresponding to an approximate value $22639''\cdot5$ of the principal elliptic coefficient, then the principal elliptic coefficient according to the Airy period (1750–1851) is

$$22639''\cdot54+\tfrac{1}{2}\alpha+\tfrac{1}{2}\beta,$$

and according to the Hansen period (1847—1901) it is

$$22639''\cdot46+\tfrac{1}{2}\alpha+\tfrac{1}{2}\beta.$$

We know that the theoretical terms $g\pm\Omega$, $g\pm\mathrm{A}'$ when A′ denotes the argument of the 273 year *Venus* term ($\mathrm{A}'-30°\cdot2$ is Professor Newcomb's A defined numerically—lxiv. p. 421), combine with terms in $\sin\Omega\cos g$, $\sin\mathrm{A}'\cos g$; while terms in $\sin g$ in both cases cancel out. There are undoubtedly other long-period terms which theory has not yet accounted for, and we may expect the same rule to hold good for these terms as well. Now if means for seventeen periods at a time be taken in the coefficients of $\sin g$ in vol. lxiv. p. 415 and Table IX. of this paper, and the coefficients of $\sin g$ thereby cleared of terms $g\pm\Omega$, $g\pm(\omega-\omega')$, $2\mathrm{D}-g+3\mathrm{V}-3\mathrm{E}$, &c., it will be found that the values thus smoothed exhibit no trace of long-period inequalities, thus confirming the analogy with the theoretical long-period terms.

23 continued. The observed cosine term is a measure of the value of $\frac{1}{9}\Delta g$, where Δg is the error of the tabular mean anomaly. We may expect to find—and in fact I have found—traces of long inequalities in the values of $\cos g$. The observed coefficient $+1''\cdot14$ in 23 (ii.) shows how erroneous Airy's mean anomaly had become in the first half of the nineteenth century. Airy,

moreover, omits all long-period inequalities. I must hold over, for the present, the discussion of motion of the perigee.

64. The observed and theoretical evections are in close accordance. This disposes of a correction of Hansen's.

51. If the tabular coefficients of terms 24 and 107 were so altered that 24 (iii.) $+0''.70$ sin disappeared, I should expect to find 51 (iii.) $-0''.17$ sin approximately change sign, and in deducing the observed coefficient for 1875 I have taken this consideration into account.

24, 107. The argument of 107 exceeds the argument of 24 by $80°.77-4.2557(p-44)$ at the middle of my pth period of analysis. The excess is therefore 163°, 334°, 162° for the middle of the periods denoted by (i.), (ii.), (iii.) respectively. Also in periods (i.) and (iii.) one argument gains 200° upon the other, and in period (ii.) 170°; putting $2\theta = 200°$ and 170°, we have $\frac{\theta}{\sin\theta} = 1.8$ and 1.5. These data are sufficient to follow through the effect of altering the tabular coefficients to $+17''.5$ and $-0''.7$.

To 24 (i.) we have to apply corrections
$-0''.4 \sin -0''.7 \div 1.8 \sin(\text{arg.} +163°) = -0''.1 \cos.$

To 24 (ii.)
$-0''.4 \sin -0''.7 \div 1.5 \sin(\text{arg.} +334°) = -0''.8 \sin +0''.2 \cos.$

To 24 (iii.)
$-0''.7 \sin.$

To 107 (i.)
$-0''.7 \sin -0''.4 \div 1.8 \sin(\text{arg.} -163°) = -0''.5 \sin +0''.1 \cos$

To 107 (ii.)
$-0''.7 \sin -0''.4 \div 1.5 \sin(\text{arg.} -334°) = -0''.9 \sin -0''.1 \cos.$

To 107 (iii.)
$-0''.7 \div 1.8 \sin(\text{arg.} -162°) = +0''.4 \sin +0''.1 \cos.$

The application of these corrections to the observed coefficients reduces them considerably. Of course the concluded observed values are not of the same order of accuracy as in the more simple cases. Nevertheless, the concluded value $+17''.5$ is extremely discordant from theory. There is moreover a cosine term as well. I can only refer to my remarks on lxiv. p. 692.

128. A similar remark to 51.

70. A term with characteristic e', where Hansen's coefficient is preferable to Delaunay's.

127. A similar remark to 140. Period, nearly one year.

25. A similar remark to 70. Characteristic ee'^2.

28. There is a suspicion of an empirical term $+0''.3 \sin(\text{arg.} -\phi)$ where ϕ (in degrees) = Year − 1775 approximately. Of course I do not consider the term as demonstrated.

29. Again a suspicion of a small empirical term.

32. Again a suspicion of a small empirical term.

112, 114. The argument of 112 exceeds the argument of 114 by $320°{\cdot}42+1°{\cdot}4889\,(p-44)$ at the middle of my pth period of analysis. The excess is therefore 292°, 358°, 58° for the middle of the periods denoted by (i.), (ii.), (iii.) respectively. Also the relative motion is so slow that $\frac{\theta}{\sin\theta} = 1{\cdot}0$ approximately. If the tabular coefficients be altered to $-1''{\cdot}0$ and $-0''{\cdot}5$, then the following corrections must be applied :

To 112 (i.)
$-0''{\cdot}1 \sin -0''{\cdot}5 \sin(\text{arg.} -292°) = -0''{\cdot}3 \sin -0''{\cdot}5 \cos.$

To 112 (ii.)
$-0''{\cdot}1 \sin -0''{\cdot}5 \sin(\text{arg.} -358°) = -0''{\cdot}6 \sin.$

To 112 (iii.)
$-0''{\cdot}1 \sin.$

To 114 (i.)
$-0''{\cdot}5 \sin -0''{\cdot}1 \sin(\text{arg.} +292°) = -0''{\cdot}5 \sin +0''{\cdot}1 \cos.$

To 114 (ii.)
$-0''{\cdot}5 \sin -0''{\cdot}1 \sin(\text{arg.} +358°) = -0''{\cdot}6 \sin.$

To 114 (iii.)
$-0''1 \sin(\text{arg.} + 58°) = \quad -0''{\cdot}1 \cos.$

The application of these corrections to the apparent observed coefficients reduces them considerably. In particular the $+0''{\cdot}46 \cos$ in 112 (i.) is accounted for. The coefficient of $\sin(g-\mathfrak{L})$ is the same as that of $\sin(g+\mathfrak{L})$ with its sign changed.

142, 143, 144. The argument of 143 exceeds the argument of 142 by $233°-2°{\cdot}414\ (p-44)$ at the middle of my pth period ; the argument of 143 exceeds the argument of 144 by $320°+1°{\cdot}520\,(p-44)$; for the middle of periods denoted by (i.), (ii.), (iii.) the excess of 143 over 142 is 280°, 173°, 76° respectively ; the excess of 143 over 144 is 292°, 357°, 58° respectively ; and consequently the excess of 144 over 142 is 348°, 176°, 18° respectively. The values of $\frac{\theta}{\sin\theta}$ for periods (i.) and (iii.) are 1·2 for 142, 143 ; 1·1 for 143, 144 ; 1·7 for 142, 144 ; for period (ii.) the same quantities are 1·1, 1·0, 1·4. With these data the effect of altering the tabular coefficients to

$$-0''{\cdot}4,\ +0''{\cdot}2 \text{ and } -6''{\cdot}6 \sin -0''{\cdot}7 \cos$$

may be followed through. We have to apply the corrections

To 142 (i.)
$-0''{\cdot}4 \sin +(0''{\cdot}2 \div 1{\cdot}2) \sin(\text{arg.} +280°) - (0''{\cdot}7 \div 1{\cdot}7) \cos(\text{arg.} +348°)$
$= -0''{\cdot}5 \sin -0''{\cdot}6 \cos.$

To 142 (ii.)
$-0''.4 \sin + (0''.2 \div 1'.1) \sin (\text{arg.} + 173°) - (0''.7 \div 1'.4) \cos (\text{arg.} + 176°)$
$= -0''.6 \sin + 0''.4 \cos.$

To 142 (iii.)
$-0''.1 \sin + (0''.2 \div 1'.2) \sin (\text{arg.} + 76°) + (0''.7 \div 1'.7) \sin (\text{arg.} + 18°)$
$= +0''.3 \sin + 0''.3 \cos.$

To 143 (i.)
$+0''.2 \sin - (0''.4 \div 1'.2) \sin (\text{arg.} - 280°) - (0''.7 \div 1'.1) \cos (\text{arg.} - 292°)$
$= +0''.7 \sin - 0''.6 \cos.$

To 143 (ii.)
$+0''.2 \sin - (0''.4 \div 1'.1) \sin (\text{arg.} - 173°) - (0''.7 \div 1'.0) \cos (\text{arg.} - 357°)$
$= +0''.5 \sin - 0''.7 \cos.$

To 143 (iii.)
$+0''.2 \sin - (0''.1 \div 1'.2) \sin (\text{arg.} - 76°) + (0''.7 \div 1'.1) \sin (\text{arg.} - 58°)$
$= +0''.5 \sin - 0''.5 \cos.$

To 144 (i.)
$-0''.7 \cos - (0''.4 \div 1'.7) \sin (\text{arg.} - 348°) + (0''.2 \div 1'.1) \sin (\text{arg.} + 292°)$
$= -0''.2 \sin - 0''.9 \cos.$

To 144 (ii.)
$-0''.7 \cos - (0''.4 \div 1'.4) \sin (\text{arg.} - 176°) + (0''.2 \div 1'.0) \sin (\text{arg.} + 357°)$
$= +0''.5 \sin - 0''.7 \cos.$

To 144 (iii.)
$+0''.7 \sin - (0''.1 \div 1'.7) \sin (\text{arg.} - 18°) + (0''.2 \div 1'.1) \sin (\text{arg.} + 58°)$
$= +0''.8 \sin + 0''.2 \cos.$

These corrections reduce the quantities in columns 3 and 4 considerably, but $-0''.4 \sin$ is still left as a residual in 144 (iii.). I must acknowledge the help I have received from Mr. Nevill's paper (lxiv. p. 588), and it will be noticed that the theoretical term $-0''.3 \cos 8$ does not seem to be consistent with the observations.

I conclude with a list of errata, one in Hansen's tables, the rest in my papers.

Hansen's Tables de la Lune, p. 30, line 5 from bottom, *for* 69·1 *read* 169·1.

In consequence of the above (*Monthly Notices*, lxiv. p. 163) coefficient of $\sin(-g - 2g')$ *for* +0·709 *read* 1·18, *for* 0″·475 *read* 0″·0. This error has been corrected in term 81 of Table X., where the comparison of observation with theory led to its discovery. The erratum in Hansen is a mere printer's error, and does not go further in his tables.

Other Errata.

lxiv.	415.	Period 98, last column.		*For* −6 *read* +6.
„	571.	Reference No.	1.	*For* + 124″·68 *read* − 124″·68.
„		„	4.	*For* −2370·10 *read* +2370·10.
„	572.	„	6.	*Read* − 24·48.
„		„	34.	*Read* + 0·316.
„		„	13.	*For* 0·42 *read* 1·02 (last column). Delete "term omitted by Airy."

lxiv. 572. Reference No. 16. *For* 0·38 *read* 0·68 (last column). Delete "term omitted by Airy."
" " 30. *For* 0·43 *read* 0·73 (last column). Delete "term omitted by Airy."
" 691. *For* argument 2ω *read* $180° + 2\omega$.

Mean Areas and Heliographic Latitudes of Sun-spots in the Year 1903, *deduced from Photographs taken at the Royal Observatory, Greenwich, at Dehra Dûn (India), and in Mauritius.*

(*Communicated by the Astronomer Royal.*)

The results here given are in continuation of those printed in the *Monthly Notices*, vol. lxiii. p. 464, and are deduced from the measurements of photographs taken at the Royal Observatory, Greenwich, at Dehra Dûn, India, and at the Royal Alfred Observatory, Mauritius.

Table I. gives the mean daily area of umbræ, whole spots, and faculæ for each synodic rotation of the Sun in 1903; and Table II. gives the same particulars for the entire year 1903 and the two preceding years for the sake of comparison. The areas are given in two forms: first, projected areas, that is to say, as seen and measured on the photographs, these being expressed as millionths of the Sun's apparent disc; and next, areas as corrected for foreshortening, the areas in this case being expressed in millionths of the Sun's visible hemisphere.

Table III. exhibits for each rotation in 1903 the mean daily area of the whole spots (corrected for foreshortening), and the mean heliographic latitude of the spotted area, for spots north and for spots south of the equator; together with the mean heliographic latitude of the entire spotted area, and the mean distance from the equator of all spots; and Table IV. gives the same information for the year as a whole, similar results for 1901 and 1902 being added, as in the case of Table II. Tables II. and IV. are thus in continuation of the similar tables for the years 1874 to 1888 on pp. 381 and 382 of vol. xlix. of the *Monthly Notices*, and for the years 1889 to 1902 on pp. 465 and 466 of vol. lxiii.

The rotations in Table I. and Table III. are numbered in continuation of Carrington's series (*Observations of Solar Spots made at Redhill*, by R. C. Carrington, F.R.S.), No. 1 being the rotation commencing 1853 November 9. The assumed prime meridian is that which passed through the ascending node at mean noon of 1854 January 1, and the assumed period of the Sun's sidereal rotation is 25·38 days. The dates of the commencement of the rotations are given in Greenwich civil time, reckoning from mean midnight.

TABLE I.

No. of Rotation.	Date of Commencement of each Rotation.	No. of Days on which Photographs were taken.	Mean of Daily Areas. Projected. Umbræ.	Projected. Whole Spots.	Projected. Faculæ.	Corrected for Foreshortening. Umbræ.	Corrected for Foreshortening. Whole Spots.	Corrected for Foreshortening. Faculæ.
	1902. d							
659	Dec. 31·09	27	4	31	225	3	25	228
	1903.							
660	Jan. 27·42	27	21	127	535	17	102	611
661	Feb. 23·77	24	18	100	396	14	85	455
662	Mar. 23·08	26	85	542	769	66	425	762
663	Apr. 19·36	27	42	270	877	30	199	1053
664	May 16·60	27	5	42	574	4	33	602
665	June 12·81	25	64	352	1018	42	249	1037
666	July 10·00	23	42	234	1261	30	175	1379
667	Aug. 6·21	27	34	183	1371	21	122	1480
668	Sept. 2·46	26	17	95	769	17	93	846
669	29·73	27	188	1438	813	145	1188	959
670	Oct. 27·02	27	224	1422	1192	171	1080	1401
671	Nov. 23·33	27	138	854	1312	103	656	1535

TABLE II.

Year.	No. of Days on which Photographs were taken.	Means of Daily Areas. Projected. Umbræ.	Projected. Whole Spots.	Projected. Faculæ.	Corrected for Foreshortening. Umbræ.	Corrected for Foreshortening. Whole Spots.	Corrected for Foreshortening. Faculæ.
1901	359	14	41	23	8·6	29	29
1902	349	14	86	163	10	62	178
1903	350	67	434	870	51	339	969

TABLE III.

No. of Rotation.	Date of Commencement of each Rotation.	No. of Days on which Photographs were taken.	Spots North of the Equator. Mean of Daily Areas.	Spots North of the Equator. Mean Heliographic Latitude.	Spots South of the Equator. Mean of Daily Areas.	Spots South of the Equator. Mean Heliographic Latitude.	Mean Heliographic Latitude of Entire Spotted Area.	Mean Distance from Equator of all Spots.
	1902. d			°		°	°	°
659	Dec. 31·09	27	16	22·44	9	14·92	+ 8·31	19·59
	1903.							
660	Jan. 27·42	27	70	23·43	32	18·08	+10·37	21·75
661	Feb. 23·77	24	62	19·82	23	22·26	+ 8·48	20·48
662	Mar. 23·08	26	137	21·47	288	18·97	− 5·95	19·78
663	Apr. 19·36	27	152	19·14	47	15·66	+10·95	18·33
664	May 16·60	27	24	18·07	10	25·07	+ 5·69	20·08
665	June 12·81	25	96	20·79	153	19·82	− 4·18	20·19
666	July 10·00	23	106	20·88	69	20·48	+ 4·50	20·73
667	Aug. 6·21	27	39	21·28	82	22·55	− 8·40	22·14
668	Sept. 2·46	26	65	12·38	28	17·98	+ 3·23	14·07
669	29·73	27	228	13·88	960	21·23	−14·50	19·82
670	Oct. 27·02	27	376	17·13	704	22·42	− 8·67	20·58
671	Nov. 23·33	27	359	17·79	297	21·95	− 0·20	19·68

TABLE IV.

Year.	No. of Days on which Photographs were taken.	Spots North of the Equator.		Spots South of the Equator.		Mean Heliographic Latitude of Entire Spotted Area.	Mean Distance from Equator of all Spots.
		Mean of Daily Areas.	Mean Heliographic Latitude.	Mean of Daily Areas.	Mean Heliographic Latitude.		
			°		°	°	°
1901	359	22	8·59	6·6	16·27	+2·82	10·37
1902	349	42	18·81	21	15·29	+7·48	17·64
1903	350	133	18·12	206	21·10	−5·75	19·93

The principal features of the record for 1903 are :

1. The increase in area ;—the mean daily value for umbræ, whole spots, and faculæ being in each case between five and six times as great as in 1902, showing unmistakably that the revival of activity after minimum is in progress.

2. So far as umbræ and whole spots are concerned this increase was due chiefly to the remarkable activity shown in the last three months of the year, the mean daily area for the first nine months being less than one-half of that for the whole year.

3. The faculæ were less unevenly distributed through the year than the spots, the second half of the year being about twice as prolific as the first half.

4. Comparing the whole spots of the two hemispheres, the area for the northern has been to that of the southern as 39 to 61. This is contrary to the precedent of the last two cycles ; for in 1880 and 1881, and again in 1890 and 1891, when the solar activity was increasing, a marked superiority was shown by the northern hemisphere.

5. Neither hemisphere has been absolutely undisturbed for a complete rotation at any time during the year.

6. The number of days without spots in 1903 was 56, as compared with 248 in 1902 and 289 in 1901. The last day without spots in 1903 was September 22.

7. The distribution of spots in latitude has been very characteristic of a new cycle when in full progress. The chief activity has been in the zone 15° to 25° in both hemispheres, whilst the belt of activity from 7° to 10°, recognised in 1902, has been almost quiescent.

8. As compared with the corresponding years of the two preceding cycles the mean daily spotted area appears rather low when taken into consideration with the mean distance from the equator of all spots. In 1880 the area was 416, and mean distance from equator 19°·85 ; and in 1891 the corresponding values were 569 and 20°·31.

9. The number of separate groups of spots was more than four times as great as in 1902, being 150 in all, of which 60 were in the northern hemisphere and 90 in the southern.

Royal Observatory, Greenwich :
1904 *December* 6.

Observations of the Leonid Meteors of 1904 November at the Royal Observatory, Greenwich.

(*Communicated by the Astronomer Royal.*)

A considerable number of Leonids were observed on the morning of November 15 (civil reckoning). The following table shows the number counted by two observers during the watch :

	G.C.T.	Number of Leonids.		G.C.T.	Number of Meteors.
d	h d h		d	h h	
Nov. 14	$22\frac{1}{2}$—15 0	5	Nov. 15	$3\frac{1}{2}$—4	52
	h h				
15	0—1	11		4 —$4\frac{1}{2}$	78
	1—2	18		$4\frac{1}{2}$—5	65
	2—3	22		5 —$5\frac{1}{2}$	58
	3—$3\frac{1}{2}$	11		$5\frac{1}{2}$—$6\frac{1}{2}$	22

The paths of the meteors were plotted till $3\frac{1}{2}^h$, but from that time the meteors were counted only, so that the numbers possibly include a few not Leonids. Fog began to obscure the sky before $4\frac{1}{2}^h$, so that the time of maximum is not determined with certainty. The largest number counted in any five minutes was 22 from $4^h\ 5^m$ to $4^h\ 10^m$.

A very fine meteor at $4^h\ 25^m$ was noted as brighter than *Jupiter* : length of path, 40° ; duration, 3 seconds. It burst, lighting up the sky, and left a trail which lasted 114 seconds.

In the early part of the watch, before $3\frac{1}{2}^h$, 7 Leonids were noted of the first magnitude and four brighter, one leaving a trail for 60^s. The paths of 35 of the Leonids were 10° or longer.

The night of November 13–14 was foggy throughout. The night of November 15–16 was cloudy, with bright intervals till 2^h, when a thick fog set in. A few meteors were observed, including 10 Leonids, one of which was as bright as *Jupiter*. Through the fog a bright double flash was seen at $3^h\ 39\frac{1}{2}^m$, but no path could be traced.

The morning of November 17 was overcast till 5^h. Between 5^h and $6\frac{1}{2}^h$ 11 Leonids were observed.

Note on the Shower of Leonids in 1904. By W. F. Denning.

On November 14 the weather here was pretty favourable for observation in the earlier part of the night. Between $13^h\ 30^m$ and $15^h\ 45^m$ I watched the eastern region of the sky during about $1\frac{1}{2}^h$, when there were fifty-five meteors, of which thirty-three were directed from the "Sickle." I concluded that Leonids were

falling at the horary rate of twenty-five for one observer, but my outlook was somewhat limited by obstructions.

After $15^h\ 45^m$ the fog increased and further observations could not be made.

The radiant of the Leonids was at $151° + 23°$. The meteors seemed apparently less bright than those forming the shower of 1903 November 15. The minor radiants of the period were unusually active, and the two most prominent of these were at $43° + 21°$ and $143° + 37°$.

I could not assure myself that the shower was increasing in intensity during my intermittent watch, but from several reports sent in by reliable observers it appears that the maximum occurred at about 16^h, or soon after that hour, when the rate of apparition reached one Leonid per minute. This would make the display somewhat richer than an ordinary Perseid shower and about one-fourth as strong as the Leonid return of 1903 November 15.

There were, however, a considerable number of brilliant Leonids recorded by various observers between 17^h and $17^h\ 17^m$, and I have received descriptions of twelve different meteors, equal to or exceeding first magnitude, which appeared during the short interval mentioned.

On November 15 at $14^h\ 40^m$ a magnificent meteor was seen at Charmouth and Torquay (where the observers estimated it as equal to the Full Moon and four times brighter than *Venus* respectively), and it appears to have been directed from a radiant in *Aries* and to have descended from eighty-three to thirty miles along a path of eighty-two miles over the north coast of France.

The fire ball lit up the sky vividly and must have presented a splendid effect over the English Channel.

Bishopston, Bristol:
1904 December 7.

A Comparison of the A. G. Catalogue (1900·0) *for Vienna* (*Ottakring*) *with the Radcliffe Third Catalogue* (1890·0). By F. A. Bellamy, M.A.

1. The publication of vol. ii. of the second or southern series of zone catalogues of the *Astronomische Gesellschaft* affords an opportunity of comparing this differential catalogue of stars with absolute places such as are given in the Radcliffe Third Catalogue (1890·0). In this Radcliffe Catalogue special attention was paid to stars south of the equator, with a view to establish a connection with the Cape (1880) Catalogue. The Vienna zone extending from $-6°$ to $-10°$ thus falls entirely

within that region of the sky observed at the Radcliffe Observatory. Stars common to the two Catalogues are 967 in number, but, omitting fundamental stars, the number used is 923. The Radcliffe Catalogue, moreover, has a special interest for the writer, since he took a considerable share in the observations during his appointment at that observatory; and he was thus led to make the comparison briefly described in the present paper.

2. The observations for the A. G. zone were commenced on 1892 January 19, and finished on 1899 January 12; most of the observations were made during the years from 1893 to 1896. Some revision work was done after 1898. The instrument used was the Repsold meridian circle with an object-glass of 5·1 inches diameter and 5 feet focus, and 120 magnifying power. The observations were made at the Kuffner Observatory at Ottakring (Vienna).* The Radcliffe observations were made with the transit circle, which is almost identical in size and power: its description may be found in the Radcliffe volumes.

At Vienna the transits were chronographically recorded and the circle read by two microscopes 180° apart; the R.A.'s and Dec.'s in a zone usually depend upon the observations of four to six fundamental stars as given in Auwers's "Mittlere Oerter von 303 Sternen nach den definitiven Fundamental-Katalog für die südlichen Zonen der Astronomischen Gesellschaft für 1900·0."† But only forty-four stars, which fall within the Vienna zone, were used from that catalogue. The Radcliffe observations were made by the eye and ear method, and the Greenwich clock-star list was used.

3. The precessions and secular variations in the Vienna and Radcliffe Catalogues are those calculated from Struve's constants. The epoch of the Vienna Catalogue being 1900·0 (which will be the epoch for all the A. G. Southern Catalogues) and the Radcliffe 1890·0, it was first necessary to bring the positions of the 923 stars to a uniform epoch. In doing this no corrections were applied for P.M.'s; after applying them to about a hundred stars I found the effect in the mean was small, so decided to use the places uncorrected for proper motion.

Group of R.A.	Right Ascension.				Declination.				Number of Stars.
	R−V.	Diff. from Wt. Mean.	Correction to Radcliffe (from Auwers).	Diff. from Mean Corrected.	R−V.	Diff. from Wt. Mean.	Correction to Radcliffe (from Auwers).	Diff. from Mean Corrected.	
h h	s	s	s	s	"	"	"	"	
0–1	−0·068	−·044	+·015	−0·029	+0·99	+0·33	−0·01	+0·32	26
1–2	− ·057	−·033	+·012	− ·021	+ ·69	+ ·03	− ·02	+ ·01	34
2–3	− ·036	−·012	+·006	− ·006	+ ·80	+ ·14	− ·01	+ ·13	35
3–4	− ·030	−·006	−·002	− ·008	+ ·71	+ ·05	− ·02	+ ·03	26
4–5	− ·025	−·001	−·007	− ·008	+ ·65	− ·01	− ·04	− ·05	44

* See description of the instrument in *Publikationen der von Kuffner'schen Sternwarte*, Band I.

† *Berliner Astronomisches Jahrbuch*, 1901, Anhang II. p. [9].

Group of R.A.	Right Ascension. R−V.	Diff. from Wt. Mean.	Correction to Radcliffe (from Auwers).	Diff. from Mean Corrected.	Declination. R−V.	Diff. from Wt. Mean.	Correction to Radcliffe (from Auwers).	Diff. from Mean Corrected.	Number of Stars.
h h	s	s	s	s	″	″	″	″	
5– 6	·000	+·024	−·008	+·016	+·56	−·10	−·06	−·16	47
6– 7	−·022	+·002	−·007	−·005	+·74	+·08	−·06	+·02	48
7– 8	−·016	+·008	−·006	+·002	+·53	−·13	−·04	−·17	40
8– 9	−·014	+·010	−·003	+·007	+·53	−·13	−·03	−·16	43
9–10	−·038	−·014	·000	−·014	+·45	−·21	−·05	−·26	40
10–11	−·008	+·016	−·001	+·015	+·38	−·28	−·08	−·36	29
11–12	−·028	−·004	−·008	−·012	+·77	+·11	−·09	+·02	41
12–13	+·005	+·029	−·012	+·017	+·79	+·13	−·10	+·03	36
13–14	−·016	+·008	−·012	−·004	+·68	+·02	−·11	−·09	44
14–15	+·011	+·035	−·011	+·024	+·89	+·23	−·13	+·10	38
15–16	−·004	+·020	−·009	+·011	+1·04	+·38	−·15	+·23	31
16–17	−·001	+·023	−·006	+·017	+0·92	+·26	−·15	+·11	40
17–18	−·015	+·009	−·003	+·006	+·74	+·08	−·13	−·05	37
18–19	−·037	−·013	−·002	−·015	+·40	−·26	−·11	−·37	50
19–20	−·030	−·006	·000	−·006	+·47	−·19	−·06	−·25	47
20–21	−·039	−·015	+·002	−·013	+·61	−·05	−·03	−·08	40
21–22	−·035	−·011	+·006	−·005	+·39	−·27	·00	−·27	41
22–23	−·039	−·015	+·011	−·004	+·72	+·06	+·01	+·07	31
23– 0	−·035	−·011	+·016	+·005	+·75	+·09	·00	+·09	35
Weighted Means	−0ˢ·024				+0″·66				923

4. In the table I have collected the mean differences for each hour of right ascension. With regard to the mean differences −0ˢ·024 in R.A. and +0″·66 in Dec., these are in great part due to the Radcliffe Catalogue. In *Monthly Notices*, vol. lv. p. 295, Mr. E. J. Stone deduced for the mean differences between Radcliffe (1890) and Greenwich (1880) for stars within the limits of −5° and −10° declination the values

$$\Delta\alpha(\mathrm{R}-\mathrm{G}) = -0^{s}\cdot017, \quad \text{and} \quad \Delta\delta = +0''\cdot91\ ;$$

and the systematic differences V−G would thus be

$$\Delta\alpha(\mathrm{V}-\mathrm{G}) = -0^{s}\cdot007, \quad \text{and} \quad \Delta\delta = -0''\cdot25.$$

But the differences in R.A. for the separate hours, which range from −0ˢ·068 to +ˢ·011, are greater than might be expected, considering the number of stars compared ; the greatest negative quantity is −0ˢ·057. If we subtract from the results for each hour the mean value of the whole, we get quantities under "difference from mean." The mean of these, without regard to

sign, is $\pm 0^s\cdot 015$; a glance at the residuals (third column) shows that the residuals are systematically positive from 5^h to 18^h, while those for 18^h to 5^h are negative. The means of the two groups are :

$$\left.\begin{array}{ll} h\ h & s \\ 5-18 & \Delta\alpha = +0\cdot 013 \\ 18-5 & \Delta\alpha = -0\cdot 015 \end{array}\right\} \text{ Difference, } 0^s\cdot 028.$$

Apparently there are systematic differences in declination, but they are not so prominent.

5. Since the Vienna observations are based on Auwers's system these differences should be mainly due to the Radcliffe Catalogue. The corrections required by Radcliffe to reduce to Auwers's system have been recently published * and are reproduced in columns 4 and 8 in the table. Inspection shows that if the differences R—V are entirely due to Radcliffe, Auwers's corrections should be approximately doubled. Thus the mean corrections given by Auwers for

$$\left.\begin{array}{lll} h\ h & & s \\ 5-18 & \text{are } \Delta\alpha = & -0\cdot 007 \\ 18-5 & \text{,, } \Delta\alpha = & +0\cdot 005 \end{array}\right\} \text{ Difference, } 0^s\cdot 012.$$

The reason of this discrepancy is probably as follows : the Radcliffe Catalogue is known to be affected with sensible errors, depending on declination, as is apparent from Mr. Stone's paper (*Monthly Notices*, vol. lv. p. 295), and from that of Dr. Auwers on p. 34 of the work quoted. The comparison with Vienna is confined to a particular zone, $-6°$ to $-10°$, while Auwers's comparison applies to all stars south of $+50°$. Hence this seems to be a case where the systematic corrections cannot be arranged in two independent series, one varying with R.A. and another with Decl., as Auwers suggests. There should be a different set of R.A. corrections for each zone of Decl. The results given by Mr. Stone for groups of 6^h (*Monthly Notices*, vol. lv. pp. 296–7) support this conclusion.

6. But the following points were briefly examined : (*a*) *Possible Magnitude Equation.*—The Vienna observations are affected, according to Dr. L. de Ball, with an equation of $0^s\cdot 0025$ per magnitude, and the Radcliffe observations are probably also affected to some extent. It was soon seen that the joint effect on the comparison in R.A. could not be sensible, for on taking means of groups the mean magnitudes were all within 0·4 of the mean magnitude of the whole (7·3). (*b*) The mean declination of the groups does show a curious tendency to systematic difference, the mean for 6^h-18^h being $8°\cdot 15$, and for 18^h-6^h it is $7°\cdot 87$, a difference of $0°\cdot 27$; but this can scarcely be responsible for any large portion of the observed differences.

* *Astronomische Abhandlungen als Ergänzungshefte zu den Ast. Nach.* Nr. 7. Kiel, 1904.

DETACHED NEBULA IN CYGNUS
Photographed by W. S. Franks.

7. In the introduction to the Greenwich Second Ten-year Catalogue for 1890 a comparison is given for ten A. G. Catalogues. In a similar manner I have determined the mean differences $\Delta\alpha$ and $\Delta\delta$ from the 138 stars in Vienna and Greenwich Catalogues and for the thirty-six fundamental stars (Auwers). Proper motions have been applied. The following are the results :

	$\Delta\alpha$.	$\Delta\delta$.		$\Delta\alpha$.	$\Delta\delta$.	
$-6°$ to $-10°$	$-0^s\cdot01$	$-0''\cdot2$	138 stars ;	$-0^s\cdot014$	$-0''\cdot36$	36 stars.

University Observatory, Oxford.

Detached Nebula in Cygnus. By W. S. Franks.

I was much interested in Dr. Max Wolf's paper on the remarkable nebula in *Cygnus* (*Monthly Notices*, vol. lxiv. p. 838), and thought it worth while to try it with the 20-inch reflector of this observatory. The accompanying photograph was obtained on 1904 November 12, with ninety minutes' exposure, between $21^h\ 36^m$ and $23^h\ 6^m$ local sidereal time ; sky clear, but partial moonlight (Moon five days old). Another photograph was obtained simultaneously with the 5-inch camera, but, as it corroborates Max Wolf's in every respect, it is not necessary to reproduce it also. The scale adopted is $1^{mm} = 30''$ of arc ; the extent of field shown is $1°\ 22'$ from p to f and $57'$ from n to s, the centre of plate being roughly in R.A. $21^h\ 49^m\cdot6$, Decl. $+46°\ 48'$ (1900). [The scale of Max Wolf's picture does not quite conform to the description, being only one-third instead of one-half of the present one ; 1^{mm} on that is therefore equal to about $90''$, not $60''$.] Owing to the superior defining power of the reflector the detail is here more clearly shown than on the former plate, though the exposure was only ninety minutes as against four hours. Although it bears a family likeness to the "trifid" nebula in *Sagittarius* it is more complicated in structure ; and, situated as it is in such a remarkably void region, it becomes a very interesting object. I have often noticed the curious thinning out of stars in the immediate vicinity of nebulæ, and undoubtedly there must be some physical cause to account for the fact, of which Sir W. Herschel was well aware. Is it possible that some of these objects are surrounded by *dark* and relatively cool nebulous matter, which, viewed in its greatest darkness round the edge, is sufficient to absorb and obliterate small stars behind it ? We have no ground for assuming that the nebulæ generally are more distant than the stars ; indeed, from their vast apparent size they may be much nearer. Considering, too, how few of the stars show any sensible parallax it may be that some of the nebulæ, when they are seriously attacked,

will yield positive results. The long barren channel preceding this detached nebula (only part of which is visible on the scale of this plate) is a very curious feature, and offers an inviting field for speculation.

Starfield Observatory, Crowborough Beacon.

Dark Nebulosities. By W. S. Franks.

From time to time it has been hinted in a vague manner that, besides the ordinary self-luminous nebulosity, there exist in space certain forms of *non-luminous* nebulous matter, but, so far as I know, nothing definite has been advanced to account for such a phenomenon. However, that it does exist there can be little doubt after examining the evidence; I propose, therefore, to mention certain objects which exhibit this curious appearance. During the course of a considerable experience in nebular photography I have met with not a few of such, but have merely selected four of them as being good typical examples. As will be seen from the illustrations figs. 1, 2, and 3 belong to the "ray"-like nebulæ, of which so many were detected by Lord Rosse; they are generally lenticular in section, and in all probability are spirals seen edgewise. A considerable proportion of these are characterised by a *dark* line or stripe along the major axis, apparently dividing the nebula into two parts parallel to each other. What is the cause of this appearance? That it is due to a real division of the nebula is almost inconceivable. It must be, I venture to assert, because the outer and attenuated edge of the nebula has cooled down so rapidly, relatively to the central hot and luminous mass, that where it crosses the latter (as it would do in an edgewise view) it simply stops its light by absorption and produces a dark band. I call it "dark" nebulosity advisedly, for it is really *darker* than the surrounding sky. This is even more strikingly seen on the negative (where it is, of course, *lighter* than the sky background). Its effect on the photographic plate is as though a screen were interposed between it and the sky, to stop the diffused light which always fogs the plate in a degree proportional to the length of exposure. In fig. 4 we have a modification of the usual form, the section being that of a double concave lens; but the dark band is there also, probably because the feeble outer nebulosity is densest along its centre. Possibly, too, some of the peculiar dark gaps which are shown in certain nebulæ of irregular form may be in reality enormous real extensions in the line of sight of feebler and cooler nebulosity which intercept the light behind. [The figures are all to one scale: $1^{mm} = 12'' \pm$ of arc.]

Starfield Observatory, Crowborough Beacon.

Fig. 1.

H V 19 Andromedæ.

Fig. 2.

H V 24 Comæ.

Fig. 3.

H I 43 Virginis.

Fig. 4.

H V 8 Leonis.

EXAMPLES OF DARK NEBULOSITIES.
Photographed by W. S. Franks.

On the Decline of the Visual Magnitude of Variable 1904 *Pegasi as observed at the Radcliffe Observatory, Oxford.*

By Walter Wickham.

The *Zirkular Nr.* 68 *der Zentralstelle*, announcing the discovery by Mr. A. Stanley Williams of an apparently new star preceding B.D. 29°·4655, was received at Oxford on October 8, and the same evening the star was found in the place indicated, using the Barclay 10-inch equatorial.

The following four B.D. stars are conveniently situated for comparison, but are so nearly of an equal magnitude that it has not been easy to formulate a descending scale from them. The magnitudes here set down are those of (1) the original B.D., (2) the Cambridge (England) zone of the A.G.C., (3) my own readjustment of the values after careful comparisons on October 8, 11, 13, 18.

	B.D.	Camb. A.G.C.	Radc. 1904.
B.D. + 29·4659	8·8	9·0	9·11
·4652	8·9	8·9	8·91
·4653	9·2	9·1	8·79
·4655	9·1	9·1	9·01

The whole of the field was surveyed and provisional magnitudes assigned to about twenty of the fainter stars, extending the Argelander scale in descending steps by extrapolation. The results for the Stanley Williams star were :

	Oct. 8.	Oct. 11.	Oct. 13.	Oct. 18.	
	9·48	9·53	9·48	9·41	
from	(8)	(10)	(11)	(22)	comparison stars.

The close agreement of these values showed that there was very little, if any, change in magnitude. This conclusion was confirmed in *Ast. Nach.* 3971, 174–6 by Professor Pickering's telegram of October 7, "Williams star, long period variable, considered by spectrum," and by the notification of October 8 from Herr P. Gotz, Astrophysical Observatory, at Königstuhl, that on August 6–8 the star was nearly of the same magnitude as B.D. + 29°4653 (9·2 mag.). In *Ast. Nach.* 3973, 207–8, there appeared a note from Professor E. C. Pickering, "The Harvard photographs show that this object has existed for several years and is variable." Professor Max Wolf has published a chart of the region of the variable in *Ast. Nach.* 3977, 267–8, but does not mention the brightness, either visual or photographic, of the star on October 9, when the photograph was taken from which is reproduced the sketch he publishes.

The whole staff of this Observatory being engaged in other work of a routine character, and the clear nights so rare, no further attention could be given to this star until December 3,

when I found that it had decreased in brightness by a whole magnitude. No opportunity for observation has since been lost, and the determinations are as follows:

	Dec. 3.	Dec. 5.	Dec. 7.	Dec. 8 (sky becoming foggy).	
	10·50	10·44	10·56	10·55	
from	(12)	(15)	(17)	(6)	comparison stars,

with a redetermination of their magnitudes.

As previously intimated the scale on which these estimations have been made will probably require revision when standard photometric values of the brightness of the comparison stars have been published elsewhere; but it seemed advisable, in view of this large change of brightness, to enlist the attention of observers who can follow the star with effective instruments should it continue to decrease to the point of visual extinction.

The colour has been red throughout, increasing as the star diminished, as was the case in the early changes of *Nova Persei*. Photographs taken with the 24-inch refractor of this Observatory on October 8 and December 5 show the change of photographic effect very markedly.

Radcliffe Observatory, Oxford:
1904 *December* 8.

On the Relative Brightness of Binary Stars.

By J. E. Gore, F.R.A.S.

In my *Catalogue of Binary Stars*, published by the Royal Irish Academy in 1891, I gave the "relative brightness" and "hypothetical parallax" for all the binary stars for which orbits had then been computed. The "relative brightness" was calculated from Mr. Monck's formula, the standard star being ξ *Ursæ Majoris*, of which the brightness was assumed as unity. Since that time numerous other orbits have been computed. From all these orbits I have selected those which seem to be the most accurately determined, and have recomputed the "relative brightness" and "hypothetical parallax" of forty-eight binary stars. My former catalogue contained fifty-nine stars, but for some of these the orbits have not proved satisfactory, and better orbits have been computed for others. I have adopted Dr. See's orbit for ξ *Ursæ Majoris*, and with his values of P, the period in years, and a, the semi-axis major in seconds of arc, Mr. Monck's formula becomes

$$k = 10^{0\cdot4(3\cdot86-m)}\left(\frac{P}{60\cdot0}\right)^{\frac{1}{3}}\left(\frac{2\cdot508}{a}\right)^{2}.$$

I have used the Harvard photometric magnitudes of the stars, and have given the spectrum as observed at Harvard, A being the first or Sirian type, F intermediate between first and second types, G second or solar type, and K intermediate between second and third types

If M be the mass of a binary system, and p the parallax,

$$M = \frac{a^3}{p^3 P^2}.$$

Putting M = 1, we have

$$p = \frac{a}{P^{\frac{2}{3}}},$$

which is the "hypothetical parallax" on the assumption that the mass of the system is equal to the mass of the Sun. From this it follows that the smaller the parallax the larger the mass, and the greater the parallax the smaller the mass.

Binary Stars.

No.	Star.	R.A. 1900·0. h m	Dec. 1900·0. ° ′	Mag.	P Years.	a.	Relative Brightness.	$h.p.$	Spectrum.	Computer of Orbit.
1	Σ 3062	0 1	+57 53	6·10	104·6	1″·37	0·89	0″·06	F	See
2	η Cassiopeiæ	0 43	+57 17	3·64	500	11·4	1·36	0·18	F8G	Comstock
3	γ² Andromedæ	1 57·8	+41 51	5·00	55·0	0·34	16·37	0·024	A	Hussey
4	Σ 228	2 7·6	+47 2	6·03	88·7	0·98	1·49	0·05	F	Gore
5	40(o²)Eridani	4 10·7	− 7 49	9	180·0	4·79	0·01	0·15	...	Doolittle
6	55 Tauri	4 14·2	+16 18	6·86	200	0·85	2·73	0·025	E	Hussey
7	OΣ 82	4 17·1	+14 51	6·54	97·9	0·94	1·06	0·04	H	Hussey
8	β 883	4 44·6	+10 52	6·5	15·8	0·24	1·62	0·04	...	Lewis
9	Sirius	6 40·7	−16 35	−1·58	51·1	7·77	12·61	0·56	A	Zweirs
10	Castor	7 28·2	+32 6	1·58	346·8	5·75	16·08	0·116	A	Doberck
11	Procyon	7 34·1	+ 5 29	0·48	40·0	5·84	2·41	0·50	F5G	See
12	9 Argûs	7 47·1	−13 38	5·30	23·3	0·61	18·04	0·074	E	Burnham
13	ζ Cancri	8 6·5	+17 57	4·71	60·0	0·86	3·90	0·056	F	See
14	Σ 3121	9 12·0	+29 0	7·26	34·0	0·67	0·287	0·06	E	See
15	ω Leonis	9 23·1	+ 9 30	5·55	116·2	0·88	4·11	0·037	E	See
16	φ Ursæ Maj.	9 45·4	+54 32	4·54	99·7	0·32	64·62	0·015	A	Doberck
17	ξ Ursæ Maj.	11 12·9	+32 6	3·86	60·0	2·50	1	0·163	G	See
18	OΣ 234	11 25·4	+41 51	6·99	77·0	0·34	4·08	0·02	E (?)	See
19	OΣ 235	11 26·7	+61 38	5·38	66·0	0·83	3·47	0·05	F	Hussey
20	γ Centauri	12 36·0	−48 25	2·38	88·0	1·02	39·08	0·05	A	See
21	γ Virginis	12 36·6	− 0 54	2·91	194·0	3·99	5·60	0·119	F	See
22	42 Comæ Ber.	13 5·1	+18 4	4·47	25·5	0·64	2·85	0·074	F	See
23	25 Can Ven.	13 33	+36 48	4·92	184	1·13	8·26	0·035	A	See

No.	Star.	R.A. 1900·0. h m	Dec. 1900·0. ° ′	Mag.	P Years.	*a.*	Relative Brightness.	*h.p.*	Spectrum.	Computer of Orbit.
24	Σ 1785	13 44·6	+29 29	7·26	199·2	″ 2·55	0·22	″ 0·07	...	Biesbroeck
25	α Centauri	14 32·8	−60 25	0·06	81·1	17·70	0·99	0·94	{G K5M}	See
26	OΣ 285	14 41·7	+42 48	7·24	97·9	0·34	4·65	0·016	...	Biesbroeck
27	ξ Boötis	14 46	+19 31	4·64	148·4	5·00	0·41	0·17	G	Biesbroeck
28	η Cor. Bor.	15 19·1	+30 39	5·13	41·6	0·86	1·43	0·076	F	Comstock
29	μ^2 Boötis	15 20·7	+37 41	6·67	219·4	1·26	1·63	0·034	...	See
30	OΣ 298	15 32·5	+40 8	6·69	56·6	0·88	0·55	0·06	...	Celoria
31	γ Cor. Bor.	15 38·5	+26 37	3·93	73·0	0·73	13·02	0·04	A	See
32	σ Cor. Bor.	16 10·9	+34 7	5·43	370·0	3·82	1·15	0·074	E	See
33	ζ Herculis	16 37·5	+31 47	3·00	35·0	1·43	3·29	0·134	G	See
34	β 416	17 12·1	−34 53	5·94	34·5	2·13	0·098	0·20	...	Gore
35	Σ 2173	17 25·2	− 0 58	5·26	46·0	1·14	0·93	0·09	F (?)	See
36	μ′ Herculis (BC)	17 42·6	+27 47	9	45·0	1·39	0·02	0·108	...	See
37	τ Ophiuchi	17 57·6	− 8 11	4·88	230·0	1·25	9·38	0·033	F	See
38	70 Ophiuchi	18 0·4	+ 2 31	4·07	88·4	4·55	0·42	0·229	K	See
39	99 Herculis	18 3·2	+30 33	5·26	64·5	1·28	1·16	0·08	F	Doberck
40	ζ Sagittarii	18 56·2	−30 1	2·71	21·6	0·57	14·02	0·07	A2F	Doberck (190
41	γ Cor. Aust.	18 59·6	−37 12	4·26	152·7	2·45	2·51	0·085	F8G	See
42	OΣ 400	20 6·9	+43 39	7·14	79·7	0·57	1·35	0·03	II (?)	Hussey
43	β Delphini	20 32·9	+14 15	3·72	27·7	0·67	5·63	0·07	F5G	See
44	4 Aquarii	20 46·1	− 6 0	6·03	129·0	0·73	4·41	0·028	A	See
45	δ Equulei	21 9·6	+ 9 36	4·61	5·7	0·28	1·74	0·09	F	Hussey
46	τ Cygni	21 10·8	+37 37	3·82	36·5	0 94	3·80	0·085	F	Burnham
47	κ Pegasi	21 40·1	+25 11	4·27	11·4	0·42	2·62	0·08	F5G	See
48	85 Pegasi	23 56·9	+26 33	5·82	24·0	0·89	0·38	0·107	E	See

Taking the mean of each class of spectrum we have the following results :

Spectrum.	Mean Relative Brilliancy.			Spectrum.	Mean Relative Brilliancy.		
A	21·81	From	8 orbits	F8G	1·93	From	2 orbits
A2F	14·02	„	1 „	G	1·42	„	4 „
E	4·40	„	7 „	H	1·06	„	1 „
F	3·05	„	12 „	K	0·42	„	1 „
F5G	3·55	„	3 „				

Here we have an almost regular decrease in relative brightness from spectrum A to spectrum K.

Omitting *Sirius*, *α Centauri*, and *Procyon*, which seem to be exceptionally near stars, we have the following results for the "hypothetical parallax:"

Spectrum.	Mean *h.p.*	Spectrum.	Mean *h.p.*
	″		″
A	0·044	F8G	0·133
E	0·057	G	0·156
F	0·070	K	0·229
F5G	0·075		

showing a regular increase in the "hypothetical parallax" from spectrum A to spectrum K.

Notes on the above List.

1. Σ 3062. Components about 6·4, 7·0; colours yellowish and bluish white.

2. η Cassiopeiæ. Components 4 and 7; colours yellowish and purple. O. Struve found a parallax of 0″·154, and a parallax of 0″·3743 was found by Schweizer-Socoloff. With Struve's parallax mass of system = 1·6222 × Sun's mass.

3. γ^2 Andromedæ (B.C.) Components 5, 5·7; bluish. Orbit very eccentric and inclination high.

5. 40 (o^2) Eridani (B.C.) = Σ 518. Components 9, 10·8; yellow, orange. Gill found a parallax of 0″·166, Hall 0″·223. With Gill's parallax mass of system = 0·6963 × Sun's mass.

7. 02 Σ 82. Photographic magnitude 6·54. Components 7, 9.

9. Sirius. Components −1·58 and 10; white, yellow. Hussey says Zweir's orbit is "the most reliable that has yet appeared." From this orbit and Gill's parallax of 0″·37 we have mass of system = 3·5465 × Sun's mass. According to Auwers the ratio of the masses is 1 : 2·119. This gives for the masses 2·4094 and 1·1371 in terms of the Sun's mass. The companion being faint in proportion to its mass the relative brightness is evidently very small, and therefore the "relative brightness" of Sirius itself is probably considerably higher than that given in the table.

10. Castor. Components 1·99, 2·85; greenish. The brighter component is a spectroscopic binary with a relatively dark companion. Period 2·98 days. A parallax of 0″·198 was found by Johnson. This would make the mass of the system = 0·2042 × Sun's mass.

11. Procyon. Companion about 13 mag.; purple. Elkin found a parallax of 0″·266, and afterwards 0″·325. The first parallax would give a mass of 6·6, and the second a mass of 3·627 times the Sun's mass. Procyon has a proper motion of 1″·245 in the direction of position angle 214°·6.

12. 9 Argûs. Components 5·7, 6·3; yellow. Burnham says that his orbit represents recent measures satisfactorily. According to Auwers the star has a proper motion of 0″·351 in the direction of position angle 195°·4.

13. ζ Cancri (AB). Components 5·5, 6·2; yellow.

14. Σ 3121. Components 7·2, 7·5; white, yellowish.

15. ω Leonis = Σ 1356. Components 6, 7; yellow.

16. ϕ Ursæ Majoris. Components 5·5, 5·5; yellowish. The "relative brightness" is unusually high and the "hypothetical parallax" very small. With any larger parallax the mass of the system would be less than the Sun's mass.

17. ξ Ursæ Majoris. Components 4·41, 4·87, Harvard. This is the standard star used in the calculations of "relative brightness." A number of orbits have been computed, but in all the periods lie between fifty-eight and sixty-three years, and in most of them between sixty and sixty-two years. See says

"the orbit of ξ Ursæ Majoris is practically all that can be desired in the present state of double star measurement," and he thinks that the parallax may be comparatively large. With the computed hypothetical parallax I find that the Sun would be reduced to 4·00 magnitude, or about the same brightness as the star. According to Lewis the mass of the companion is greater than the mass of the primary star in the ratio of 3 : 2 (*The Observatory*, 1904 November). ξ Ursæ Majoris is also a spectroscopic binary, so that the system is really a triple one.

18. OΣ 234. Components 7, 7·8; yellowish.

19. OΣ 235. Components 6, 7·8; yellowish. The orbit is somewhat doubtful according to Hussey.

20. γ Centauri. Components 3, 3; yellowish.

21. γ Virginis. Components 3·65, 3·68, Harvard. Orbit good. Belopolsky found a parallax of 0″·051 and a mass equal to fifteen times the Sun's mass.

22. 42 Comæ Berenicis. Components 5·2, 5·2; orange. Orbit satisfactory.

23. 25 Canum Venaticorum. Components 5, 8·5; white, blue.

24. Σ 1785. Components 7·3, 7·5; pale yellow, bluish. Orbit published in 1904.

25. α Centauri. Components 0·36 (spectrum G) and 1·61 (spectrum K 5 M), Harvard. Both orange yellow. A parallax of 0″·75 was found by Gill, and 0″·76 by Wright and Palmer from spectroscopic measures of relative velocity. With Gill's parallax mass of system = 2·00 × Sun's mass. Masses of components nearly equal. If we take the density of companion as 2·0 (Sun = 1·4), I find luminosity of primary = 2·52 × luminosity of companion or relative brightness = 0·39.

26. OΣ 285. Components 7·5, 7·6; yellowish, whitish. Spectrum not in the Draper Catalogue.

27. ξ Boötis. Components 4·7, 6·7; yellow, purple.

28. η Coronæ Borealis. Components 5·5, 6; yellowish.

29. μ^2 Boötis. Components 6·7, 8; white.

30. OΣ 298. Components 7, 7·4; yellowish. Hussey says: "Celoria's period is not far from the truth."

31. γ Coronæ Borealis. Components 4, 7; yellow, blue.

32. σ Coronæ Borealis. Components 6, 7; yellow, bluish.

33. ζ Herculis. Components 3, 6; yellow, bluish. Lewis thinks that the primary star is probably a binary with a period of about twelve years, and $a = 0{\cdot}25$ (*Monthly Notices R.A.S.* vol. lxi. No. 2). This would give a relative brightness for the primary star of 25·82, which seems improbable.

34. β 416. = Lacaille 7215. Components 6·4, 7·8; yellowish. Spectrum not in Draper Catalogue.

35. Σ 2173. Components 6, 6; yellow.

36. μ^1 Herculis. Components 9·4, 10; bluish white, bluish. Orbit good. The relative brightness is unusually small, and the hypothetical parallax is comparatively large for so faint a star. Components possibly gaseous.

37. τ Ophiuchi. Components 5, 6; yellowish.

38. 70 Ophiuchi. Components 4·5, 6·0; yellow, purplish. Kruger found a parallax of 0″·169 and 0″·162, and Schur 0″·16. With Schur's parallax the mass of the system would be 2·939 × Sun's mass; but, using the same parallax, M. Adalbert Prey finds the masses of the two components to be 0·32 and 1·28 × Sun's mass; that is, mass of fainter component is four times that of the brighter! (*Popular Astronomy*, 1904 June and July).

39. 99 Herculis. Components 6, 11·7; yellow, purple.

40. ζ Sagittarii. Components 3·9, 4·4; yellowish.

41. γ Coronæ Australis. Components 5, 5; yellowish. Gore's orbit (1892) gives P = 154·41 years and $a = 2''{\cdot}55$, differing but little from See's result.

42. OΣ 400. Components 7·5, 8·5. Spect. II.? Not in Draper Catalogue. Burnham's period is 81·04 years; $a = 0''{\cdot}47$.

43, β Delphini. Components 4, 6; yellowish. Several other orbits have been computed with periods ranging from 22·97 to 30·91 years.

44. 4 Aquarii. Components $6\frac{1}{2}$, $7\frac{1}{2}$; yellow.

45. δ Equulei. Components 5, 5·5; yellow. Hussey finds mass of system = 1·89 × Sun's mass and parallax = 0″·071, from spectroscopic measures.

46. τ Cygni. Components 4, 10. Burnham thinks the orbit doubtful and suggests that the companion is double, and on this hypothesis he has based his orbit.

47. κ Pegasi. Components 4·5, 5·0; yellowish. The brighter component is a spectroscopic binary. From the spectroscopic measures of 24·8 miles a second and period of six days I find a probable mass of the system = 0·48 × Sun's mass, and a parallax of 0″·106. This would reduce the Sun to a star of about 4·87 magnitude.

48. 85 Pegasi. Components 6, 10; yellowish, bluish. Brunnow found a parallax of 0″·054. This, with See's orbit, gives the mass of the system = 7·77 × Sun's mass. From measures of the binary and a distant 9th magnitude star Comstock, using a parallax of 0″·04, finds a mass of 11·3, the mass of A being 4·3 and that of B = 7·0 in terms of the Sun's mass = 1. With reference to this "remarkable" result he says, "A star A whose spectrum is of the second type (E in the Draper Catalogue) emits more than 100 times the light of its companion B, although B is presumably of equal age with A, and possesses 60 per cent. more mass than the latter star" (*Astrophysical Journal*, vol. xvii. p. 223). The companion is possibly gaseous. 85 Pegasi has a large proper motion of about 1″·3 per annum in the direction of position angle 140°.

On the Relative Efficiency of Different Methods of Determining Longitudes on Jupiter. By A. Stanley Williams.

I have read Professor G. W. Hough's paper "On the Determination of Longitude on the Planet *Jupiter*" in the Supplementary Number of the *Monthly Notices* very carefully, but fail to see any reasons for modifying the conclusions come to in my previous communication on this subject, in the March number. It seems to me that much of what he says has no direct bearing upon the subject and only tends to obscure the real questions at issue, and there are, moreover, several misstatements and misconceptions on his part. The position is simply this: In my previous communication I discussed in a particular manner not merely a few selected observations of a limited number of spots, but a very large amount of work done by Professor Hough with the micrometer upon numerous spots, and in addition I included his fine series of observations of the red spot, made when that remarkable object was at its maximum plainness. The first was to show the degree of accuracy attained by the micrometric method in general work, and the last what could be accomplished under the most favourable circumstances. I also discussed *in exactly the same manner* a correspondingly large number of observations of many different spots by the method of transits, made by various observers, and I came to the conclusion, as the result of this *exactly similar comparison*, that the apparent errors of the observations are about the same with either method, or, in other words, that the two methods

yield practically results of equal accuracy.* Now in order to prove that this conclusion is erroneous it is necessary to show either that the mode of discussing or comparing the observations employed by me unduly favours the method of transits at the expense of the micrometric method or else that the selection of observations by the former method made by me is not a fair one, and hence does not really represent the average work of careful observers. And I submit that Professor Hough has not done either of these things, or in fact even weakened in the slightest degree the force of my conclusions.

For firstly, with regard to the mode of discussing the observations, it matters little if at all for our purpose whether we take the "probable error," or the "mean error," or the "mean residual," or any other form of expressing the apparent "error," or of indicating the accordance of the observations. The only point of real importance is that *exactly the same kind of "error"* must be used in discussing the observations made by both the two methods in order to properly and fairly compare the results obtained, and this is what I have done in my previous paper.† I selected for the purpose in view not the "average residual" but the "mean error" of an observation as defined in § 26 of Airy's work on *The Theory of Errors of Observations*; and every result included in my paper is expressed in this form of "error," with the single exception of the first one in the list on p. 433 of the *Monthly Notices*. I am perfectly aware that Schmidt gave $\pm 1^m\cdot 9$ as the mean error of his corrected observations of the red spot, instead of $\pm 1^m\cdot 5$, as stated in my paper; but the former is not the "mean error" as defined by Airy in § 26 of his work.‡ If Professor Hough will work out the "mean error" of Schmidt's corrected observations in the manner described by Airy on p. 19 of his book above cited, I think that he will find that it is $\pm 1^m\cdot 5$, as stated in my previous paper.

* The figures given by Professor Hough on p. 824 (paragraph 4) of his paper, it should be remarked, are not my figures, nor are the conclusions my conclusions.

† I should have thought that this must be obvious to any one, and there cannot be two questions about the matter. Professor Hough apparently considers that I had accused him of deriving the errors in his 1882 Report in an unusual or incorrect way. This is not, however, the case. What I disputed, and what I dispute now, is the legitimacy of comparing the very small errors found by him in a particular way with the necessarily much larger errors derived for the method of transits, or for that matter which would be derived also for the micrometric method, in a different manner altogether.

‡ The same remark will apply to the mean errors of Dr. Lohse's observations, referred to on p. 825 of the *Monthly Notices*. There are reasons why the Potsdam observations might be expected to show somewhat larger residuals, for they were not always made in the same way. Sometimes a micrometer was used, sometimes not, whilst a few observations by other observers (Kempf and Müller) were included. They do not form a perfectly homogeneous series. Some of the observations do not seem to have been central meridian transits at all. The mean error, as previously defined, of Schmidt's uncorrected observations would be less than $\pm 3^m\cdot 0$.

The same thing will apply to Professor Hough's "corrected" value for Professor Barnard's observations of the red spot, 1901, on p. 824 of the *Monthly Notices*, and to other differences instanced by him on that page, though I do not understand how some of his figures have been arrived at.* The mean error of Jedrzejewicz's observations is certainly $\pm 0^m{\cdot}5$. It is doubtful if the use of a micrometer in the manner employed by this observer is of any advantage. In every case, with the single exception referred to above, the values given in my lists are correct, and Professor Hough is not justified in altering them.

Secondly, as regards the selection of the observations by the method of transits, Professor Hough affirms that "the table of so-called data consists almost entirely of the comparison of five or six observations *inter se*, selecting such as show a small mean residual." The first part of his statement, so far as it is correct, applies also to the results of his own micrometric observations. The last part of it is, however, quite incorrect. What I actually did was to search through all the likely periodical literature readily accessible to me for a good many years back, and I included in my lists and published *every instance* come across in my search, except as mentioned below, *entirely regardless* whether the residuals were small or large. The only cases rejected were, as mentioned in my paper, a few in which there were only three or (in one case only) four observations of a spot, and where consequently the mean error could not be ascertained with any exactness. I submit, therefore, that the data published in the lists in my paper represent *the fair average work* of careful observers in general by the method of transits, particularly if the adverse circumstances referred to in § 12 of the above-mentioned paper are borne in mind. The observations were certainly not selected on account of the smallness of the residuals. Is Professor Hough quite correct, though, in saying that "Barnard, without any investigation, *thought* some of his observations would have a mean error $\pm 0^m{\cdot}7$"? In *Pub. A. S. P.*, vol. i. p. 91, Professor Barnard makes the definite statement that from twenty-three transits of the red spot "I get for the transit of the middle from the mean of the nine observations the error of the transit $= \pm 0^m{\cdot}7$." I did not, however, as stated by Professor Hough, incorporate Barnard's data in my table, but assumed a larger mean error ($\pm 1^m{\cdot}0$). With this single

* For instance, Professor Hough states (p. 824) that my result for the red spot 1900 "carries a mean residual $\pm 2^m{\cdot}8$, *not* $\pm 0^m{\cdot}8$." But the actual residuals given by my observations are, $-0^m{\cdot}2$, $-0^m{\cdot}1$, $+1^m{\cdot}2$, and $-0^m{\cdot}9$ (*A. N.* 3675). The mean error is $\pm 0^m{\cdot}8$, as correctly stated in my paper, the mean residual being $\pm 0''{\cdot}6$. Professor Hough's figures for the mean error differ so frequently from mine that some additional data contained in his lists on pp. 828 and 829 of the *Monthly Notices* cannot possibly be accepted as comparable without examination. As this would entail the calculation of the rotation periods of the spots and the computation of the residuals shown by the observations this work must be deferred.

exception the values given in my lists are correct and cannot be questioned.

It doubtless would not be difficult to select from the large number of published observations by the method of transits, by observers good, bad, and indifferent, and sometimes with quite small telescopes, instances of large or apparently variable errors, and this no doubt will account for some of the discordances that have been referred to. Even in the case of the comparatively small number of published micrometer observations it is not difficult to pick out such instances. For example, the mean error, as previously defined, of Professor Hough's twenty-five observations of the red spot in 1894–5 * is as large as $\pm 3^{m}\cdot 5$, and there are residuals as great as $-10^{m}\cdot 0$ and $+11^{m}\cdot 6$. In other words, there is an extreme difference of $21^{m}\cdot 6$ in the time of transit of the red spot as deduced from the micrometer measures! It should be mentioned here that I purposely omitted this result from the list on p. 431 of the *Monthly Notices*, because it seemed to me that it would not be fair to the micrometric method to include it, since Professor Hough remarks that the observations were "comparatively rough, for the reason that the spot was often too indistinct to see any definite outline." On the other hand I did include the comparatively large mean errors for my own observations of this spot in the years 1898 to 1902, because I did not wish to reject a single instance come across in my search above alluded to, in which the observations were published in sufficient detail to enable the mean error to be readily ascertained, notwithstanding that the spot was then not only exceedingly faint, but there was the added difficulty that the altitude of *Jupiter* was very low. This is not the only instance in which the micrometric method was knowingly unduly favoured at the expense of the method of transits. But selected instances really prove very little, and there is now sufficient material available in order to compare general results.

With regard generally to the question of the inclusion of results derived from spots observed on a few nights only, to which Professor Hough makes objection, if we had a sufficient number of numerously observed spots we could dispense with them. An extremely small mean error—less, say, than $\pm 1^{m}$—is in such cases no doubt to some extent fictitious or accidental, but other spots similarly observed on only a few occasions may quite obviously give similar fictitious or accidental *large* mean errors. But by taking the average result of a sufficiently large number of *spots* the effect of such fictitious or accidental errors will be eliminated ; in the long run the fictitious small errors will be balanced by the fictitious large ones. That this has been the case very nearly in the present instance will be apparent if we make a little analysis of the data in the March number of the *Monthly Notices*.

* See *A. N.* 3354.

If we divide the data in question into two classes, treating all spots with less than ten observations as having a small number of observations, and those with ten or more observations as having a large number, we shall get the following figures :

Average Mean Error. Method of Transits.

	Small No. of Obs. m	No. of Cases.	Large No. of Obs. m	No. of Cases.
Red spot	± 1·8	3	± 1·7	13
Other spots	± 1·9	12	± 2·0	6

Average Mean Error. Micrometric Method.

	Small No. of Obs. m	No. of Cases.	Large No. of Obs. m	No. of Cases.
Other spots	± 1·8	10	± 2·5	5

In forming the first result I have omitted the first item in the list on p. 433 of the *Monthly Notices*, as this has been questioned. It will be seen that the spots with only a small number of observations give very nearly the same average mean error as the spots with a large number of observations so far as the method of transits is concerned. But—and this is somewhat curious—in the case of Professor Hough's micrometric measures the average mean error from spots with only a few observations is decidedly smaller than that shown by the spots with a large number of observations. In other words, the effect of including the spots with a small number of observations has been to *reduce* the magnitude of the average mean error in the case of the micrometric method, so that Professor Hough in objecting to the inclusion of such instances has actually been unwittingly damaging his own case! So far as the method of transits is concerned it makes practically no difference if the spots with only a small number of observations are omitted. It is interesting to contrast the position taken up here by Professor Hough with respect to fictitiously small errors deduced from a few observations with his treatment of the small residuals shown by his few *Saturn* observations in the *Monthly Notices* for January and April of this year. Yet the probabilities are vastly greater that the small residuals of the latter are "fictitious" than they are in any of the cases included in my lists, even if it were certain that all his observations related to the same spot.

Surely Professor Hough is not speaking seriously when he says (p. 825) that "if Schmidt's corrections were valid . . . the corrected observations can no longer be regarded as eye-estimates"? I cannot believe that this is really what he meant to say. No doubt these corrections are of an empirical nature, but they certainly do diminish the magnitude of the

residuals considerably, whilst the observations are numerous enough to give grounds for accepting them. I quite fail, however, to see why a constant error varying with the position of the planet should necessarily "show the untrustworthy nature of eye-estimates." Surely not if we can ascertain its value at any time. Can Professor Hough point out any instance of a similar correction varying with the hour angle and affecting the observations of any other observer? It would seem that if the explanation he suggests for the origin of this error is correct, then its effects, so far at any rate as regards its variable part, should be eliminated by always observing with the eyes parallel to the belts. In this connection it seems proper to inquire whether Professor Hough always makes his measures with his head in the same invariable position with respect to the belts? Otherwise the experience of those engaged in astrographic work would seem to show that appreciable errors might creep in. It seems desirable that Schmidt's observations should be rediscussed from the point of view of a rotation period varying slightly with the time, in the manner adopted by Professor Hough in reducing his own observations. Such a rediscussion might materially reduce the apparent errors of the uncorrected observations and throw additional light on this correction question. Schmidt seems rather to have tried to force his observations to fit in with a uniform period.

As already stated selected instances prove very little, and they cannot modify the conclusions derived from the comparison of a mass of material, such as that on which those contained in my previous paper are based. On p. 827 of his paper Professor Hough has considered more particularly my observations of the red spot in 1887 in conjunction with others made by Mr. Denning during the same opposition. He appears to consider these observations as showing that the method of transits is not capable of yielding results of precision. It seems to me, however, with all deference to his opinion, that the only things really important brought out by the comparison is the danger that may arise from combining the observations of two different observers, and how unfair it may be to one observer to discuss his results mixed up in this way with those of another, so that I welcome this opportunity of considering the matter further, especially since it affords a good illustration of the small importance usually possessed by selected individual instances.

In the first place I would point out that nearly all my observations are weighted with the lowest possible weight 1 (on a scale ranging from 1, bad, to 5, good)—that is, they were expressly stated to be *bad observations*. In fact the seeing in the spring and summer of 1887 was often very unsteady at Brighton, and this, combined with the faintness of the red spot, and in particular of its preceding end, rendered it difficult to secure satisfactory observations of its transit with the $6\frac{1}{2}$-inch

telescope used.* These observations of mine are, therefore, no more adapted to indicate the accuracy attainable by the method of transits than are Professor Hough's own observations of 1894–5 so suited with regard to the micrometric method. Next, in order to show the unfairness of discussing the observations of one observer mixed up with those of others, I give below my own observations apart from those of Mr. Denning. The weights on the scale, 1–5, have been added from p. 96 of my *Zenographical Fragments*, but otherwise the figures have been taken from Professor Hough's paper, excepting that the residuals O − E have been determined afresh.

1886.	Williams Longitude. m	Wt.	O−E. m
Dec. 20	−11·0	1	+2·0
1887.			
Jan. 1	− 6·0	1	+7·0
Feb. 26	− 4·8	1	+8·2
Mar. 20	− 3·9	1	+9·1
„ 27	− 6·9	1	+6·1
„ 29	−15·8	1	−2·8
Apr. 3	− 9·9	2	+3·1
„ 10	−21·8	1	−8·8
„ 17	−12·7	2	+0·3
„ 20	−13·8	3	−0·8
„ 24	−15·2	2	−2·2
„ 27	−14·9	1	−1·9
„ 29	−17·9	1	−4·9
May 2	−15·6	1	−2·6
„ 14	−15·1	1	−2·1
June 19	−21·0	1	−8·0
	Mean error =		±4·4

1886.	Denning Longitude. m	O−E. m
Nov. 23	− 2·5	+6·9
Dec. 17	− 4·9	+4·5
1887.		
May 7	−15·5	−6·1
„ 9†	−15·5	−6·1
„ 10	−13·8	−4·4
„ 26	−14·8	−5·4
June 10	−11·3	−1·9
„ 22	− 6·1	+3·3
July 16	− 5·8	+3·6
Aug. 6	− 4·2	+5·2
	Mean error =	±4·7

The mean error of an observation is here $\pm 4^m\cdot 4$, not $\pm 4^m\cdot 7$, as given by Professor Hough on p. 827 of the *Monthly Notices*, or $\pm 4^m\cdot 9$ on p. 829, but it is evident that the observations are not well satisfied by Marth's ephemeris ; there is a preponderance of plus residuals in the first half of the series and of minus ones

* In *Popular Astronomy* for 1903, p. 69, Prof. Hough states that "my measures, when the spot was very indistinct, have been referred to t he centre of the bay" of the S. equatorial belt. In light of this statement it seems probable that the Dearborn observations of 1887, published on p. 827 of the *Monthly Notices*, do not relate to the red spot at all, but to the deep bay or hollow in the S. equatorial belt—a different feature altogether.

† This observation is attributed to W. D. in the table in Professor Hough's paper, but it was actually made by D. W. only observed the following end of the spot.

in the last half. A slight shortening of the rotation period corresponding to the ephemeris would considerably reduce the magnitude of these residuals, poor though the observations admittedly are.* It will be seen that *no less than* 12 of the observations have the lowest weight 1, and also that the four observations with higher weights give much smaller residuals, the mean error in fact of these four observations being only $\pm 1^m{\cdot}6$, slightly less than that given by the Dearborn observations.

Mr. Denning's observations are given separately on the right side of the above table. The mean error is here $\pm 4^m{\cdot}7$, but with regard to these observations it should be remarked that the first two must have been made under most disadvantageous conditions, when the planet was quite close to the Sun and its altitude very low. So probably were the last two or three observations.† Now there is a simple and it seems to me good reason why these observations made under such unfavourable circumstances should show plus residuals, and this reason is the faintness of the preceding end of the red spot in 1887. This is referred to on p. 96 of my *Zenographical Fragments*, and it was there pointed out that, owing to this peculiarity, there would be a tendency for the apparent centre of the spot to shift towards the following end in confused seeing; in other words, the transits might be observed later under such unfavourable conditions. In 1893 the spot had a somewhat similar appearance, and there is some evidence of a shift of this kind occurring in bad seeing.‡ If this view is correct, and it seems to me to be a reasonable one, we have a sufficient explanation of the occurrence of these plus residuals in the early and late observations. It is clear, moreover, that in this case if micrometer measures had been made by Mr. Denning at these times they likewise would have been similarly affected. The figures on p. 828 of the *Monthly Notices* quoted as given by me for the different rotation periods of the red spot in 1887 are quite correct, but it should be remarked that these figures were partly obtained by comparing the observations of two different observers, and they should be rejected.§ The Brighton observations taken alone do not indicate these supposed

* With a somewhat shorter rotation period than that of the ephemeris the mean error can, by a graphical process, be reduced to $\pm 3^m{\cdot}2$. The period of the ephemeris is $9^h\ 55^m\ 40^s{\cdot}63$, but the rotation period of the red spot in 1887 seems to have been less than $9^h\ 55^m\ 40^s$. Professor Hough's observations would seem to make the period rather too long.

† Mr. Denning has himself called attention to the fact that observations made at such times are often necessarily of a rough or uncertain character, though it is useful to have them sometimes, owing to the long time interval they afford. See *Astronomical Register*, vol. xxii. p. 183.

‡ See *Memoirs B. A. A.* vol. iii. p. 139.

§ The writer would be extremely obliged if those who possess his little work *Zenographical Fragments* would kindly strike out the last lines on p. 96 referring to the irregular motion of the red spot, adding a reference to the foregoing.

changes of period, neither do Mr. Denning's if the above explanation of the origin of the plus residuals in his early and late observations be accepted.* In any case a few discordant observations of a very faint spot made under what must have been very unfavourable conditions could not be held to prove them. Even when my book was written I was very doubtful as to the reality of the changes in question, and used the qualifying words "according to the observations" in referring to them. Professor Hough's remark that "here we have apparently a well-established fluctuation in the rotation period of 5·2 seconds which did not exist" therefore loses all sense. With my present knowledge of the different manner in which different observers may regard the same planetary marking I should not dream of coming to any such conclusion from the published observations, or from a comparison of the observations of two different observers, even if they related to a conspicuous spot. It is apparently from cases like the foregoing, based on the comparison of 16 observations of a very faint spot, 12 of which are clearly stated to be bad ones, by one observer, mixed up with 10 observations, some of which must have been of a very rough character, by another observer, and without making any adequate investigation himself into the circumstances, that Professor Hough largely bases his condemnation of the method of transits! His observations of the red spot in 1894–5 have already been alluded to, and I give them below for comparison with the above. They have been taken from his paper in the *Astr. Nach.* 3354, but the residuals O—E have been added by the writer.

1894.	Hough Longitude. m	O—E. m
Sept. 26	+ 1·1	− 3·5
Oct. 10	− 5·4	−10·0
Dec. 3	+ 9·1	+ 4·5
„ 8	+ 6·8	+ 2·2
„ 13	+11·2	+ 6·6
„ 25	+16·2	+11·6
„ 28	+ 4·3	− 0·3
1895.		
Jan. 2	+ 2·2	− 2·4
„ 9	+ 8·4	+ 3·8
„ 16	+ 1·8	− 2·8
„ 23	+ 5·4	+ 0·8
„ 26	− 2·6	− 7·2
„ 28	+ 0·5	− 4·1

1895.	Hough Longitude. m	O—E. m
Feb. 2	+3·1	−1·5
„ 14	+6·1	+1·5
„ 16	+5·6	+1·0
„ 19	+1·7	−2·9
„ 21	+4·9	+0·3
„ 23	+6·8	+2·2
„ 26	+8·9	+4·3
Mar. 5	+1·8	−2·8
„ 10	+5·7	+1·1
„ 19	+9·4	+4·8
Apr. 3	−0·7	−5·3
„ 10	+3·9	−0·9
	Mean error	= ±3·5

* In answer to an inquiry respecting his observations of 1887 Mr. Denning writes that "my observations of the red spot in 1887 were very unsatisfactory, owing to the faintness of the object and to the bad definition

As already stated the mean error of an observation is here $\pm 3^{m}\cdot 5$, whilst there are residuals amounting to $-10^{m}\cdot 0$ and $+11^{m}\cdot 6$. The grouping of the residuals is also interesting and instructive in comparison with the foregoing observations of 1887. Both at the beginning and end of the series are rather large minus residuals; whilst the curious way in which plus or minus residuals are repeatedly grouped together suggests that the observer several times in the course of the observations suddenly changed his habit of observing. If the first two observations had not been made a different conclusion might have been come to respecting the rotation period of the spot in 1894–5. I do not think that my observations of 1887 can be said to compare unfavourably with these ones of Professor Hough, which latter were only said to be "comparatively" rough. It is easy to see, moreover, that if only the two observations at the commencement and the two at the end had been made, with a few of the intermediate ones, the result might have resembled Mr. Denning's series pretty closely, with reversed signs.

Until we are able to compare contemporaneous series of observations by several observers using the micrometric method,* it seems to me idle and useless to enter into the question of personal equation, or of what Professor Hough calls "variable error," so far at least as regards comparing the efficiency of the results obtained by the two methods of observation is concerned. "Variable error of the method of transits" has a formidable look about it, no doubt, but in most of the cases instanced on pp. 829 and 831 it would be perfectly legitimate to change the signs and call it the "variable error of the micrometric method," since even with our present knowledge a similar error undoubtedly does sometimes affect observations made by the latter method. Both personal equation and this "variable error" beyond all doubt arise in great measure from different observers not observing irregular markings in the same manner, and I can see no reason at present for supposing that they would be smaller in the case of the micrometric method than they actually are with the method of transits. It is notorious how differently different observers see planetary markings, even the most prominent ones, and it is absurd to imagine that they would all measure them alike. Even in the case of such an unusually large, definite, and regular marking as the red spot when at its maximum plainness a glance through the numerous

usually prevailing. For these reasons my transits obtained in that year were about the very last I should have chosen as a fair example of accuracy in eye-estimated transits . . . they were some of the worst observations I have ever made."

* Since the appearance of a spot is dependent largely on the aperture of the telescope employed and the atmospheric conditions prevailing, such micrometric work ought to be undertaken not only by different observers at different stations, but also with telescopes of various sizes, before any fair comparison can be made with the results obtained by different observers by the method of transits.

drawings of the period will show, I venture to think, that differences of the kind must necessarily arise. Some observers, for instance, draw the red spot at this time neatly rounded off at the ends; others show the ends pointed; others with the ends not only pointed, but drawn out into fine lines; yet others with the ends dissimilar; and so on. Dr. O. Lohse once saw the spot with the following end rounded off, but with the preceding end pointed and unsymmetrically flattened, the point being, moreover, extended into a fine line. I doubt if any two observers would agree as to what constituted the middle of such an object, the appearance of which might vary largely with the conditions of the seeing and the size of the telescope used. Professor Hough has himself drawn the following half of the red spot much broader than the preceding half. How would he measure such an object? Would he measure a point midway between the two ends, or would he measure what appeared to be the centre of figure of the spot? Again, a very common form of spot on *Jupiter* takes the shape of a dark mass projecting obliquely from a dark belt into one of the bright zones. Such spots are frequently exceedingly dark and definite at the projecting part, and from thence fade off gradually. Probably again no two observers would agree as to what is the middle of such a spot, and whilst some would no doubt try to observe the middle, or the spot, as one mass, others, including the writer, would probably observe the more definite little projecting part, though this might be far removed from the centre of the mass. On a poor night this more definite projecting part might be obliterated, and then the last-mentioned observers would necessarily observe the spot as one mass, or its apparent middle. This is probably one and a very common way in which what Professor Hough calls "variable error" arises; but micrometer measures would assuredly be affected in the same manner. It is not difficult to see how differences amounting to 5 or even 10 minutes in the time of transit might occur in this manner; and, as every spot may be said to have its own peculiarities of appearance, personal equation or "variable error" originating in this manner might be expected to differ in the case of different spots, as is actually the case. From the Dearborn observations of the red spot of 1894–5 it would not be difficult to pick out two or three "variable errors." Another instance of the kind may be referred to as occurring in Professor Hough's famous early series of observations of the red spot. In 1881 he observed the red spot in a pretty uniform manner for a couple of months up to December 28, when a sudden change seems to have taken place in his habit of observing, and during 1882 January he systematically observed the transits, as deduced from his micrometer measures, nearly 4 minutes earlier. These, and other cases that could be adduced, show clearly that, even according to our present knowledge, Professor Hough must be wrong in assuming that the micrometric method is free from this "variable error."

Referring again to these Dearborn observations of 1894–5 of the red spot, it is possible that they may furnish the key to the origin of some part, though only some part, of this "variable error." They seem to indicate that perhaps observers may not always have been careful to distinguish between the centre of figure and the centre of mass, or intensity, of the red spot, and may sometimes have observed the one and sometimes—particularly in poor seeing—the other.

Much importance seems to be attached to the accordance of measures made on the same night before and after the central transit of a spot; but although this, if correct, would show that the same observer on the same night, and after the lapse of only an hour or two, may measure a spot in the same manner, yet it does not prove in the least that another observer, with another telescope and at another place, with different conditions as to seeing, must measure the same spot in the same manner, nor even the first observer on a different night and under different conditions. It has no bearing upon this question at all. Such accordance would indicate that the disc of the planet was exactly bisected, but would the measures show that the *spot* was similarly bisected? But I do not think that Professor Hough is justified in making the statement as to the accordance of the measures made before and after the transit of a spot. In the 1882 *Report of the Director of the Dearborn Observatory* particulars are given of the measures of the red spot. On six nights there are not less than two measures on either side of the central meridian. These give the following differences for the time of transit of the red spot:

	m		m
1880 July 22	Diff. = +0·7	1880 Dec. 31	Diff. = −2·0
Sept. 9	,, = −2·7	1881 Aug. 9	,, = +2·0
Nov. 1	,, = −0·4	Dec. 28	,, = +0·4

The *plus* sign signifies that the measures made when the spot was following the central meridian make the time of transit later than those made before the transit. It would seem from the above figures that differences amounting to two or three minutes in the time of transit of a spot may have arisen according as to whether the measures were made when it preceded or followed the central meridian. The material is not sufficient to decide if there is any systematic error, but the grouping of the *plus* and *minus* differences is suggestive of this.

On p. 830 of the *Monthly Notices* the very remarkable statement is made that "although the motion of no spot on *Jupiter* can be regarded as constant for a long interval, yet there is never any abrupt change in the drift, as indicated by my observations for the past twenty-five years." Can Professor Hough have forgotten his observations of a white equatorial spot in 1881–2 (see the *Report of the Director of the Dearborn Observatory*,

1882, p. 51)? Concerning this spot he remarks that "during 1881 the single spot, observed continuously for a period of 252 days, indicated *sudden deviations* in its apparent place . . . the comparison with the ephemeris shows a maximum displacement of sixteen minutes of time."* The italics are mine. As a matter of fact it is chiefly in the equatorial regions of *Jupiter* that "sudden deviations" of this kind occur, according to observations made by the method of transits, and it can only be because Professor Hough has not apparently observed a large number of these equatorial spots in a sufficiently continuous manner that he has not frequently come across similar deviations revealed by his micrometer measures. These deviations or wanderings, as I have termed them, are familiar to every systematic observer of the equatorial markings. They were first pointed out, I believe, by Herschel in 1780,† and their existence led this distinguished astronomer to abandon the planet *Jupiter* for *Mars*, so far as the chief purpose he had in view at the time was concerned—namely, the question of determining whether the *Earth's* diurnal motion is perfectly equable.

The reason for my venturing to question the correctness of Professor Hough's identification of four observations of an equatorial spot (*Monthly Notices*, p. 833) is that the accordant observations of four different observers show that there were several similar equatorial spots near the same place, and his four observations fit in with three separate spots. As to the number of observations required to insure correct identification, opinions will, no doubt, differ, but it does not seem as though four observations of an equatorial spot scattered over an interval of eighty days is sufficient. In this particular case two of them moreover are close together at the end, so that practically the identification is dependent upon three observations, separated by thirty-seven and forty-one days. A glance over the observations of the spot referred to in the preceding paragraph will satisfy any one, I should think, that the identification might well be incorrect under such circumstances when there are several similar spots close together.

With regard to *Saturn*, since the rotation of this planet and that of *Jupiter* is performed in nearly the same time, the reduction in scale due to the greater distance and smaller size of the

* I find that Mr. Denning had already called attention to this in the *Monthly Notices* for June last, p. 768.

† *Phil. Trans.*, 1781, p. 126. These deviations or wanderings occur in the case of the neatest and most definite markings, and they cannot possibly be confounded with any of the discordances of the nature of personal equation or variable error, for the following reasons: (1) They are attested by the accordant contemporaneous observations of several different observers. (2) They may cause differences in the time of transit of a spot so considerable as to amount to more than half an hour in a few days. (3) They may affect nearly simultaneously several adjacent spots, whilst other similar spots, in the same latitude but in a different longitude, at the same time remain unaffected.

former planet must affect the accuracy of the results obtained by the micrometric method. The reduction in scale actually amounts to near one-half, and, since the Dearborn micrometrical measures of Jovian spots give an average mean error of $\pm 2^m$ in the deduced time of transit of a spot, it follows that similar measures of spots on *Saturn* may be expected to have a mean error of about $\pm 4^m$. This is, I should imagine, a minimum value, and it might be nearly twice as large in the case of an individual spot. Is there any reason why the method of transits should undergo a greater reduction in accuracy than the micrometric method? The observations of Professor Barnard, quoted on p. 832 of the *Monthly Notices*, show clearly what an indefinite and difficult object the spot must have been, but would micrometer measures of such an object have given a more accurate result than a central meridian transit? I doubt it very much.

There are several other matters considered in Professor Hough's paper that ought to be gone into, but this communication has already reached to an undue length and their consideration must be deferred. Some of them do not seem to require noticing. I need hardly add that any conclusions come to in his paper, so far as they imply any superiority on the part of the micrometric method, are not warranted by the known facts.

I must confess that I have been surprised to find that the micrometric method does not yield results of greater precision, and this circumstance, taken in conjunction with Professor Hough's own admission that his micrometer measures of Jovian spots *are only half as accurate* as those of double stars,* seems to suggest that there may be some hitherto unsuspected source of error affecting the micrometric method. Possibly the placing of a micrometer wire over a spot may prejudicially affect its appearance to an appreciable extent? A very minute spot is apt to disappear altogether under such circumstances, as is also a larger one if very faint and indefinite. In many cases it would seem as if this must give rise to the possibility of error. For instance, if the middle part of an irregular spot is cut out by a "wire" placed over it, it would seem as though errors might readily be introduced, and that such errors might even be dependent to some extent upon the thickness of the micrometer wires, so as to be different to another observer using wires of greater or less fineness. They might even vary to the same observer with the change in the apparent diameter of the planet. On p. 829 of the *Monthly Notices* reference has been made to a curious instance of an apparent difference in longitude between two spots as deduced from transits observed by Professor Barnard compared with his direct micrometer measures of the distance separating them. It does not seem, however, to have occurred to Professor Hough that the origin of this difference may possibly be traced to the micrometer. He also seems to have overlooked the fact that Barnard in his transit

* *Popular Astronomy*, 1903, p. 66.

observations used a micrometer with one wire bisecting the disc, presumably set so as to indicate the measured half-diameter of the planet, so that Barnard's *central meridian* can hardly have differed in the sense implied. Concerning the direct measures of the distance between the two spots Professor Barnard writes that "although the spots were usually very conspicuous, it was found that they were quite ill-defined and rather difficult when the wires were placed over them."* This statement seems to support the suggestion made above as to the possible prejudicial effect of a micrometer wire in altering the appearance of a spot. Without a micrometer these spots were almost ideal objects for observation.

Hove: 1904 *November* 29.

On the Eclipse of Agathocles. By Simon Newcomb.

In my "Researches on the Motion of the Moon," which appeared in 1878, I made a careful study of the accounts of supposed total eclipses of the Sun by ancient authors, with a view of determining whether any of them could be used either as tests of the lunar tables, or as auxiliaries in the determination of the slow changes in the lunar elements. The conditions required for this use were that some determinable eclipse should have been total at a known place. The conclusion which I reached was strongly in the negative. Not only did there appear to be no ancient eclipse which we could conclude was really total at a given place; but the accounts were generally so vague that no interest seemed to attach even to a comparison with the lunar tables, except for chronological or historical purposes.

I may add, in all frankness, that these adverse views have not been shared by those astronomers who, in the meantime, have made researches on the subject. Oppolzer and, after him, Ginzel had such confidence in the reality of these eclipses as to use them as the basis of corrections to the elements of the Moon's motion which are incompatible with gravitational theory. I have no intention to argue my view at present further than to say that I am not at all convinced it was in any point ill founded, so far as related to data available at the time. But since my paper was published a very important point has been brought to light showing an exception to the conclusions there reached. This arises in connection with the eclipse of Agathocles —309 August 14.

This eclipse has been so fully discussed by Airy and others that only a very brief statement of the circumstances connected

* *Monthly Notices*, vol. li. p. 549.

with it is necessary. The eclipse was observed from the fleet of Agathocles about 10 o'clock A.M. of the day after it put to sea from Syracuse. There is no doubt about the fleet having been well within the line of totality. Unfortunately it is uncertain, from historical evidence, whether Agathocles sailed to the north or to the south of Sicily. Consequently there are two possible positions of his fleet, one to the south of the island, the other near its north-east point. Moreover, the path of totality was quite broad, the radius being about 50′, thus much widening the range of uncertainty.

What gives real importance to this eclipse is its identification by Celoria with one referred to by Cleomedes, during which the Sun was said to have been entirely eclipsed in the Hellespont, while one fifth of its diameter was still visible at Alexandria. This affords a very strong presumption that the path of totality passed over the Hellespont. Yet the path does not reach the Hellespont by the tables either of Oppolzer or Ginzel, the central line being a hundred miles or more to the south. What I have now done is to make a computation of the path of totality from the tables based on the corrected theory found in the researches already alluded to, using the tables in the astronomical papers of the *American Ephemeris*, vol. i., as well as testing the result by a differential reduction of Hansen's results given in his "Darlegung." The result is that, according to my tables, the path of totality passed the following points, the one near Sicily, the other south of the Hellespont :

	Central Line.		Northern Limit.	
	° ′	° ′	° ′	° ′
Longitude ...	15 35	23 39	15 35	23 39
Latitude ...	37 7	38 1	38 4	38 58

The extreme south-west end of the Hellespont is in latitude 40°, longitude 24°. According to my corrected theory, the northern limit of the path passed some miles south of this point, and, of course, yet farther south of other regions of the Hellespont. In order to bring the northern limit well into the Hellespont it would be necessary to make a yet farther diminution of the secular acceleration by about 1″·5.

Both positions of Agathocles, as estimated by Airy, fall within the computed belt of totality, so that we cannot base any conclusion on the account of that navigator.

Very interesting in this connection is the conclusion reached by Mr. Nevill in his paper in the *Monthly Notices*, vol. xxxix. p. 72. Here the author argues vigorously against my introduction of a rather indefinitely described eclipse of the Moon used by Ptolemy ; and, indeed, of the series of Babylonian eclipses of the Moon which I used in my determination of the secular acceleration. My reply to this view would only have been that I fixed the weight to be assigned to these eclipses solely upon the expressions used by Ptolemy, and did not think it expedient to make

any change on account of the observed discordance. Possibly this was not the best course in a case where so much suspicion may attach to a record. Mr. Nevill's conclusion was that my result should have been still further diminished by at least 1″, and perhaps 1″·5. This view is now strengthened by the eclipse of Agathocles.

The important point is that this reduction will carry the observed acceleration down almost to the theoretical value, in which no allowance is made for tidal retardation. In other words, the conclusion to which the new evidence points is that the actual retardation of the Earth's rotation is almost evanescent. Although no numerical determination of the probable amount of retardation, as given by theory, has, so far as I know, ever been made, I think any estimate must make probable a value larger even than that corresponding to my former result. It therefore seems likely that a neutralisation of the effect of tidal friction is produced by some cause not yet fully investigated.